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THE

CAMBRIDGE COURSE

OF

ELEMENTARY NATURAL PHILOSOPHY:

BEING THE PROPOSITIONS IN

MECHANICS AND HYDROSTATICS,

In which those Persons who are not Candidates for Honours are examined for the Degree of B.A.

ADAPTED FOR THE USE OF COLLEGES AND SCHOOLS.

FOURTH EDITION.

To which are now added numerous Examples and Problems, from the University Examination Papers and Original, with Hints for their Solution.

By J. C. SNOWBALL, M.A.,
FELLOW OF ST. JOHN'S COLLEGE, CAMBRIDGE.

CAMBRIDGE:
MACMILLAN AND CO.
BAILY, BROTHERS, LONDON.
MDCCCL.

PREFACE.

THE following pages contain the demonstrations of the questions in the Course of Elementary Natural Philosophy in which the University of Cambridge requires the Candidates for the Ordinary Degree of B.A. to be prepared for Examination. The Enunciations adopted by the Senate have here been adhered to, and the Propositions are proved in the same order as they have been numbered by Authority.

It has been endeavoured to give such proofs and explanations as might be found intelligible and easy by Students possessing a moderate acquaintance with Arithmetic, the first principles of Algebra, and Euclid's Elements. To the present edition (the Fourth) a Collection of Examples and Problems has been added.

ST. JOHN'S COLL., CAMBRIDGE,
November, 1850.

SCHEDULE OF MATHEMATICAL SUBJECTS of Examination, for the Degree of B.A. in the University of Cambridge, of Persons not Candidates for Honours.

ARITHMETIC

Addition, subtraction, multiplication, division, reduction, rule of three: the same rules in vulgar and decimal fractions; practice, simple and compound interest, discount, extraction of square and cube roots, duodecimals.

ALGEBRA.

1. Definitions and explanation of algebraical signs and terms.
2. Addition, subtraction, multiplication, and division of simple algebraical quantities and simple algebraical fractions.
3. Algebraical definitions of ratio and proportion.
4. If $a : b :: c : d$, then $ad = bc$, and the converse:
   also $b : a :: d : c$,
   and $a : c :: b : d$,
   and $a + b : b :: c + d : d$.
5. If $a : b :: c : d$,
   and $c : d :: e : f$,
   then $a : b :: e : f$,
6. If $a : b :: c : d$,
   and $b : e :: d : f$,
   then $a : e :: c : f$.
7. Geometrical definition of proportion. (EUCLID. Book v. Def. 5.)
8. If quantities be proportional according to the algebraical definition, they are proportional according to the geometrical definition.
9. Definition of a quantity *varying as* another, *directly* or *inversely*, or as two others *jointly*.

EUCLID.

Books I. II. III.

Book VI. Props. 1. 2. 3. 4. 5. 6.

[The remaining portion of the Schedule forms the Table of Contents of this Book. The explanatory Articles to which asterisks are prefixed are not included in the University Course.]

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21. Prop. 3. If two forces acting perpendicularly on a straight lever in opposite directions and on the same side of the fulcrum balance each other, they are inversely as their distances from the fulcrum; and the pressure on the fulcrum is equal to the difference of the forces.
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THE ELEMENTS

OF

MECHANICS AND HYDROSTATICS.

The explanatory articles and paragraphs which are enclosed in brackets form no part of the University Course. Those in smaller type are illustrative of the Definitions and Propositions they immediately follow. They form a small and a very easy portion of the book, and will be found useful in solving Problems.

THE ELEMENTS OF MECHANICS.

CHAPTER I.

DEFINITIONS; EXPLANATION OF STATICAL FORCES; THE MANNER IN WHICH THEY ARE MEASURED AND REPRESENTED.

1. The science of Mechanics investigates the causes that prevent or produce motion in bodies, or that tend to produce or prevent motion.

It is divided into two parts. The one, which investigates the conditions fulfilled when a body is in a state of rest, is called Statics. The other, which treats of the causes and the effects of motion, is called Dynamics.

Thus it is the province of Statics to show how the roof of a building is supported by the beams and the walls. If the roof gave way and fell, it would belong to Dynamics to account for the circumstances attending the fall; to explain why the motion took place in one direction rather than in another, to determine the time elapsed in falling, and the swiftness of motion at any instant.

The part of Mechanics treated on in the following pages is *The Elements of* Statics.

2. DEF. Whatever the cause may be that produces or prevents motion, or that tends to produce or prevent motion, it is called a FORCE.

[If a heavy body, as a stone, be laid on the open hand, experience shows that to prevent the stone from falling the hand must make some effort. Again, to set a ball rolling along the ground requires some exertion. The effort, or exertion, is called in either case a FORCE: and although the effect produced be not great enough to prevent entirely the fall of the stone, or to communicate motion to the ball, yet it is still denominated a FORCE.

From the definition of STATICS given in Art. 1, it will readily be understood that in that branch of MECHANICS the conditions are investigated which are fulfilled by those forces only which keep a body at rest.]

3. DEFS. (1) All bodies, if left to themselves, would fall perpendicularly to the earth's surface. This power that resides in the earth, of drawing all substances to it, is called THE FORCE OF GRAVITY.

(2) The act of a body's pressing downwards to the earth when laid on any substance is called the GRAVITATION of the body.

(3) The *precise amount* of gravitation residing in any particular body is termed the WEIGHT of that body.

4. [*The* WEIGHTS *of different bodies may be thus compared.* If two bodies, when successively attached in the same manner to a spring, so that they may act upon it by their weights in the same way, produce the same effects, (by bending the spring to the same extent,) the weights of the bodies are equal. Any other body that produces the same effect on the spring by its weight as both the former bodies when applied at the same time do by their weights, has its weight double of that of either of them. And by means of such a contrivance as this spring, bodies might be shown to be three, four, or any number of times the weight of the first body.

The WEIGHT of any body is thus *measured.* The weight of a certain bulk of some particular substance is first fixed upon as a standard. Thus the weight of a piece of *lead* of a certain size being called a *pound*, any other body whose gravitation produces the same effect as four, or six, or ten such pieces of lead, will be four, or six, or ten pounds in weight, as the case may be.]

5. DEF. The substance, material, or stuff, of which any body is made, is called MATTER.

[*As all bodies have* Weight, the property of having Weight is to be considered *as necessarily* and inherently belonging to all Matter; and it is with

respect to this, its property of having Weight, that Matter is made the subject of mathematical investigation. The existence of Matter, under whatever form it may appear, is therefore tested by that form having Weight; and in the same ratio or degree that one body has more weight than another, it is concluded that it contains more matter.]

6. DEF. THE QUANTITIES OF MATTER which different bodies contain are proportional to the weights of the bodies.

[Thus, if a body A weigh *one* pound, and another body B weigh *three* pounds, the quantity of matter contained in A is *said* to be to the quantity contained in B as 1 to 3; or B is said to contain three times as much matter as A does.

The exact quantity of matter contained in any body might be measured by comparing its weight with the weight of some particular body that has been fixed upon for a standard. Thus, if a *cubic inch* of *water* be previously taken as the body to measure the quantities of matter contained in all other bodies by, and the quantity of matter in this cubic inch of water be called *one*, then the quantity of matter in any other body would properly be said to be *five*, if the weight of that body were five times as great as the weight of the cubic inch of water.]

7. DEF. When the DENSITIES of different substances are spoken of, the closeness is referred to with which the matter composing them is packed, as it were, in the substances.

To compare this closeness, let *equal* bulks, or sizes, of two different substances be taken; and suppose the substances to be Water and Lead. If the bulk of *water* which is taken weigh *one* pound, it will be found that the piece of lead of equal size with it will weigh $11\frac{4}{10}$ pounds. There is evidently, therefore, $11\frac{4}{10}$ times as much heavy matter in a piece of lead as there is in an *equal* bulk of water; and this fact is expressed or described by saying, that "The DENSITY of Lead is to the DENSITY of Water, as $11\frac{4}{10}$ is to 1;" or by saying, "The DENSITY of Lead is $11\frac{4}{10}$ times that of Water."

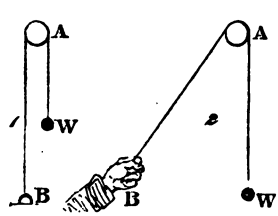
COR. 1. If the density of water be *called* 1, *i. e.* if *water* be taken to *measure* Density, then the density of lead will be properly called $11\frac{4}{10}$, or 11·4.

[COR. 2. In the same manner as it has been explained how the density of lead is estimated with respect to the density of water, the densities of any other substances, whether solid or fluid, may be determined with respect to that of water.]

8. *To explain how the precise amount of a* STATICAL FORCE *is described; that is, to show how Statical Forces are measured.*

The amount of a Statical Force is described by stating the number of pounds it would support if the Force were made to act directly opposite to the Force of Gravity. Thus if the weight of a body were P pounds, and it were prevented from moving towards the earth's surface by a hand placed beneath it, the resistance offered by the hand to the communication of motion (that is, the *force* exercised by the hand), would be P pounds; and if this same pressure were produced by the hand in any other direction, it would be described in the same manner, by saying that it "was equal to P pounds," or that it "was" "P pounds." If, therefore, in the following pages, a Force be represented by P, it is meant that P is the *number of pounds* that the Force would support.

9. [FORCES are, in Statics, also called PRESSURES. In whatever direction a force tends to produce motion, its magnitude, as has already been stated, is measured by the weight of the body which would exert the same effect to produce motion downwards, as the force under consideration exerts in the line in which *it* endeavours to produce motion. And that such a method of measuring Forces is allowable appears from *this* consideration, viz., that the effect produced by the weight of a heavy body may be made to take place in *any* direction whatever; horizontally, as in the case of a string being attached to an object lying on a table and kept stretched by a heavy body (W) that hangs over the side of the table; or vertically upwards, as in fig. 1, by passing the string over a peg A and attaching the end to a ring B, so that BA may be vertical; or in any other direction (fig. 2), by making the heavy body pull the string in a line BA, which is inclined at any angle to that (AW) in which it acts itself.



10. (1) The point at which a force acts upon a body is *called the "point of application"* of the force.

(2) The *line* in which a force produces, or tends to produce, motion, is called "*the line of the force's action*;" and *any* line that is parallel to the line of a force's action is said to be "*in the direction of the force's action*," or "*in the direction of the force*."

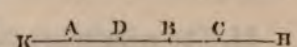
[When the *direction of a force's action*, (or, as it is generally called, "*the direction of the force*,") is indicated by a line, either the very line is given in which the force acts, or some line which is parallel to it. "*The line of a force's action*" and "*the direction of the force*" must by no means be confounded together. If the former be known, the latter is necessarily known also; but if only the latter be given, the precise line in which the force produces, or tends to produce, motion, is uncertain, and all that can be said respecting it is, that the line of action of the force is either that line or some other line which is parallel to it.]

(3) If two or more forces be applied to a body, or at some point, and no motion is produced, they are said to "*counteract*," or to "*balance*" one another, or to be "*in equilibrium*."

11. *Forces may be properly represented by lines.*

Since lines may be drawn of any length and in any direction from a point, the lines in which forces act, and the ratios which the forces bear to one another, may be represented by drawing lines which coincide with the lines in which the forces act, and whose lengths bear to one another the same ratios that the forces themselves bear to one another.

[Among other advantages which attend this method of indicating the magnitudes and directions of forces, the addition and subtraction of such forces as act at a point in the same straight line are easily effected. Thus, if a certain force act at A in the line AH, and AB be taken to represent it, and another force, half the size of the former, act at A in the same line, and also tend to move the body from A towards H, then, by taking BC equal to the half of AB, the line AC will represent the whole pressure at A, both with respect to the *magnitude* of that pressure and to *the line in which it acts*. And, in like manner, if a force equal to half the original force AB act at A in the line AH, but tends to move the body at A from A towards K, half the pressure of the former force will be counteracted by this new force. Cutting off from the line AB, therefore, a part BD equal to the half of AB, the effective pressure still remaining will be properly represented by AD, with respect both to its *magnitude* and to its *line of action*.]



12. [NOTE.—It will be gathered from the last Article, that a

force AB applied to A has not the same effect as a force BA applied at that point; for a force AB would tend to move a body at A in the line KH towards H, but a force BA would tend to move a body at A in the line KH from A towards K. It is not, therefore, indifferent whether the words "a force AB" or "a force BA" be used; since, though the two forces represented by AB and BA be the same in *magnitude*, and also act in the same straight line, yet they tend to produce motions directly opposite to one another; the force AB tending to move the body at A towards H, and the force BA tending to move the body at A towards K.]

13. [*The effect produced at a point by any force is the same at whatever point in its line of action the force is applied.*

For if a heavy body P be suspended by a string CP, the force necessary to prevent P falling to the earth is found to be the same whether that force be applied at A, or B, or C;—the weight of the string being either neglected, or the weight of that portion of it which is supported along with the heavy body P, being counterbalanced.]

14. [DEF. If a string, fastened at one end, be pulled by a force applied at the other end, the resistance to motion made by the string at any point in it is called the TENSION of the string at that point.

If the string be supposed to be without weight, it will follow from Art. 13 that the tension at *every* point of it is the same; namely, the force by which the string is pulled.]

15. [To recapitulate the substance of this Chapter.

(1) MATTER is the substance of which all bodies are composed. It is found, by universal experience, to have a tendency to move towards the earth. Art. 5.

(2) The precise amount of the pressure with which any *particular body* endeavours to move towards the earth is called the WEIGHT *of that body*.

This WEIGHT of a body is *measured* by comparing the tendency of the body to move towards the earth with that of some other given body (of a certain size and formed of a certain material) which is taken for a standard, and to which the name of a *grain*, an *ounce*, a *pound*, or a *ton* is given, as the case may be. Art. 3.

(3) THE QUANTITIES OF MATTER contained in different bodies (whether the bodies be great or small, rare like gas or dense like lead) is *proportional*, (not *equal*,) to the weights of the bodies. Art. 6.

(4) The DENSITIES of different substances are *proportional*, (not *equal*,) to the weights of *equal bulks* of the substances. Art. 7.

(5) I. Whatever be the cause which moves, or tends to move, matter existing under any form whatsoever, it is called a FORCE.

FORCES which prevent motion taking place,—that is, STATICAL FORCES,—are measured by the number of *pounds* they would support if they acted vertically upwards. Arts. 2, 8.

II. THE LINE OF A FORCE'S ACTION is the actual line in which the force tends to produce, or to prevent, motion. Art. 10. And if a force be said to be P, it is meant that it is equal to P *pounds*.

III. THE DIRECTION OF A FORCE is indicated either by the line of its action, or by any line which is parallel to the line of its action. Art. 10.

IV. The magnitudes of forces, and the lines (or the directions) in which they act, may be represented by means of straight lines. Art. 11.

V. If a straight line AB represent a force acting on a material point placed at A, and BA represent another force acting on the same point, the two forces AB and BA are equal, and tend to move the point in the same straight line, but in opposite ways from A. Art. 12.

VI. A Statical Force produces the same effect at whatever *point in its line of* action it is applied. Art. 13.

VII. To investigate the effects produced by a force there must be given:—

1st. The magnitude of the force;—which is known if the number of pounds be known which the force would support.

2nd. The point at which the force is applied.

3rd. The line in which the force acts,—or the direction; for knowing the *direction* of the force and the point it is applied at, the *line* in which the force acts may be determined by drawing a line through the given point parallel to the given direction.

[The APPENDIX contains a collection of EXAMPLES and PROBLEMS.]

CHAPTER II.

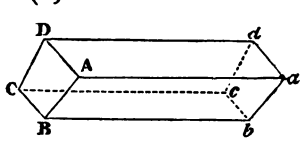
THE LEVER.

16. DEFS. (1) A PLANE, OR A PLANE SUPERFICIES, is that superficies, or surface, in which, if *any two* points be taken, the straight line joining them lies wholly within the superficies. EUCLID I.; Def. 7.

(2) A SOLID is that which hath length, breadth, and thickness. EUCLID XI.; Def. 1.

(3) PARALLEL PLANES are such as do not meet one another, however far they may be produced. EUCLID XI.; Def. 8.

(4) A PRISM is a solid bounded by plane rectilineal figures; two of them, which are parallel to each other, being similar and equal triangles, squares, or polygons, having the similar angles in each opposite, and all the remaining plane rectilineal figures which bound the prism being rectangles.

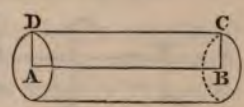


[Thus, let *ABCD* and *abcd* be two equal and similar quadrilateral *figures, placed with their planes parallel, and with their similar angles opposite to each other;* and let all the figures, such as *AB b a*, which are formed

by joining the equal angles of $ABCD$ and $abcd$, be rectangles; then the solid included by these rectangles and by the *ends*, or *bases*, $ABCD$ and $abcd$, is called a *prism*.

The *length* of the prism is any one of the edges Aa, Bb, &c.; which lines, being the sides of adjacent rectangles, are all equal to one another.]

(5) A CYLINDER is a solid described by a rectangle ($ABCD$) revolving round one of its sides (AB) which remains fixed.



[The side AB, which remains fixed, is the *length* of the cylinder, and is called its *axis*.

The surfaces described by the two sides AD and BC which are adjacent to the fixed side of the rectangle, are circles in form, and are parallel to each other; they are called the *ends* or *bases* of the cylinder.]

17. DEF. A LEVER is a rigid and inflexible rod, moveable in one plane round a point in the rod called the FULCRUM.

In the following propositions the thickness of the rod is neglected, and the lever is considered to be a geometrical line; it is also supposed to be without weight. The weights, or Forces, acting on the Lever are supposed to act in the same plane.

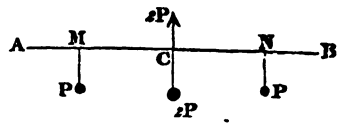
18. The investigation of the properties of the Lever will be made to depend on the following Axioms, which the mind readily admits as true.

AXIOM I. If two *equal* weights (or other forces that are exerted in the same direction) act perpendicularly upon a horizontal straight lever, they will balance round a fulcrum placed at the middle point between the points at which they are applied; the pressure on this fulcrum will be the sum of the weights, and it will act in the same direction as the weights act.

AXIOM II. If *any* two weights (or forces), acting perpendicularly on a horizontal straight lever, on opposite sides of a fulcrum, balance each other, the pressure on the fulcrum is equal to the sum of the weights.

AXIOM III. The efforts made by two *equal* forces, acting perpendicularly at the extremities of *equal* straight arms of any lever, *to turn the* lever round, are equal.

19. PROP. I. *A horizontal prism, or cylinder, of uniform density, will produce the same effect by its weight as if it were collected at its middle point.*



Let P and P be two equal weights acting perpendicularly at M and N on the straight horizontal lever AB which is moveable round any fulcrum in it. They produce the same effect as if they were applied at C, the middle point between M and N.

At C let a weight $2P$ be added, acting perpendicularly downwards, and also let a pressure equal to $2P$ act perpendicularly upwards at C. Since these two forces are equal and opposite, they will counteract each other; and whatever the tendency of the lever previously was to turn round the fulcrum, it will remain unaltered.

Now since P at M and P at N would produce a pressure downwards of $2P$ on a fulcrum at C (Axiom I.), the pressure upwards exerted by such a fulcrum would be $2P$. The pressure therefore of $2P$, which has been supposed to act *upwards* at C, will serve instead of such a fulcrum, and will therefore balance P at M and P at N. Wherefore, the three forces, P at M, P at N, and $2P$ acting upwards at C, have no tendency whatever to turn the lever about; but its tendency to turn about (which has been shown to be the same as when the lever was acted on by no other forces than P at M and Q at N) arises entirely from a force $2P$ applied at C, and acting downwards.

Now supposing * that a horizontal prism (or cylinder) of uniform density, would produce the same effect by its weight, as if the whole matter composing it were collected into a number of small equal weights placed at equal distances along a horizontal straight line of the same length as the prism (or cylinder), then, since it appears from what has been proved above, that

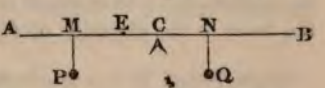
* A uniform prism, or cylinder, may be divided by means of equally distant planes which are parallel to its ends, into equal small prisms or cylinders lying on each side of the plane passing through its centre. Now, if these be supposed to be condensed and placed at equal distances along a *straight line equal* to the length of the original prism or cylinder, the supposition made in the text will not appear unreasonable.

every pair of weights which are equidistant from the center of the line would produce the same effect as if it were collected at the center, the whole prism therefore, or cylinder (being made up of such pairs of equal weights), will produce the same effect by its weight as if it were collected at its center. Q. E. D.

Cor. Hence it follows that a horizontal prism, or cylinder, of uniform density, will balance on its middle point; and the pressure on a fulcrum placed there will be the weight of the prism, or cylinder.

20. Prop. II. *If two weights, acting perpendicularly on a straight lever, on opposite sides of the fulcrum, balance each other, they are inversely as their distances from the fulcrum; and the pressure on the fulcrum is equal to their sum.*

Let there be an uniform cylinder of any length AB, and whose weight is $P + Q$. If it be placed horizontally it will balance on a fulcrum C placed at its middle point (Prop. I., Cor.); and the pressure on C will be the weight of the cylinder, namely $P + Q$.



Let AB be divided in E, so that $AE : AB :: P : P+Q$; then, since the cylinder is supposed to be of uniform density, Weight of the part AE of the cylinder : weight of the whole :: $AE : AB$

$$:: P : P + Q, \text{ by construction;}$$

but, Weight of the whole $= P+Q$; $\therefore$ weight of the part $AE = P$; and $\therefore$ weight of $EB = Q$.

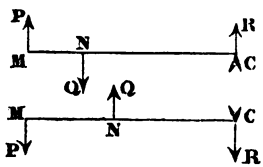
But if the parts AE and EB be collected at their middle points M and N, and act perpendicularly on the lever, they will produce the same effect by their weights as before. Prop. I. Therefore, the weights P and Q, acting perpendicularly at M and N, will balance about C, and the pressure on C will be $P+Q$. Now

$$\frac{P}{Q} = \frac{AE}{EB} = \frac{AB-BE}{AB-AE} = \frac{2BC-2BN}{2AC-2AM} = \frac{BC-BN}{AC-AM} = \frac{CN}{CM}$$

$$\text{or } P : Q :: CN : CM.$$

Wherefore, "If two weights, &c." Q. E. D.

21. PROP. III. *If two forces, acting perpendicularly on a straight lever in opposite directions and on the same side of the fulcrum, balance each other,* (1) *they are inversely as their distances from the fulcrum, and* (2) *the pressure on the fulcrum is equal to their difference.*



Let the forces P and Q, acting perpendicularly at M and N on the straight lever MC in opposite directions and on the same side of the fulcrum C, balance each other; Then $P : Q :: CN : CM$, and (Q being the force which is the nearer to the fulcrum) the pressure on the fulcrum $= Q - P$.

Let the fulcrum at C be removed, and let its resistance (R) be supplied by a force equal to R, and acting perpendicularly to the lever in the direction of P's action.

Since P and R are counterbalanced by Q, they must produce a pressure at N equal and opposite to Q. Let Q be removed, and its place supplied by a fulcrum on the contrary side of the lever to that on which Q acted, sustaining the pressure (namely Q) produced by P and R.

The case is now that of two forces acting perpendicularly on opposite sides of the fulcrum, and balancing each other; and therefore (by Prop. II.),

$$P : R :: CN : NM;$$

$$\therefore P : P + R :: CN : CN + NM :: CN : CM.$$

But Q, the pressure on the fulcrum which has been supposed to be placed at N, is (by Prop. II.) $= P + R$.

$$\therefore P : Q :: CN : CM.$$

Also since $Q = P + R$, $\therefore R = Q - P$.

Wherefore, " If two forces, &c." Q. E. D.

22. [From the last two Propositions it appears, that if a *straight lever, which* is acted on perpendicularly by two weights,

or other forces, P and Q, respectively applied at the distances CM and CN from the fulcrum C, be at rest, then, whether P and Q act on the same or on different sides of the fulcrum, the proportion $P : Q :: CN : CM$ is always true.

When the Lever is used to balance a given weight or force Q, by the application of another force P, Q is usually called "*the Weight*," and P "*the Power*."

If CM, the perpendicular distance from the fulcrum at which the Power acts, be greater than CN, the distance at which the Weight acts, the Power required to balance the Weight is less than the Weight; in this case "*force*" is said "*to be gained*" by the application of the lever. But if CM be less than CN, the Power required to balance the Weight is greater than the Weight, and "*force*" is then said "*to be lost.*"]

23. PROP. IV. *To explain the different kinds of levers.*

LEVERS are divided into three classes.

(1) Where the Power and the Weight act on *opposite* sides of the fulcrum.

(2) Where the Power and the Weight act on the *same* side of the fulcrum, but the perpendicular distance from the fulcrum at which the Power acts is *greater* than that at which the Weight acts.

(3) Where the Power and the Weight act on the *same* side of the fulcrum, but the perpendicular distance from the fulcrum at which the Power acts is *less* than that at which the Weight acts.

Of the FIRST class the poker, when used to raise the coals, is an instance; the bar of the grate on which the poker rests being the Fulcrum, the force exerted by the hand the Power, and the resistance of the coals the Weight. In the common balance,

the Power and the Weight are equal forces perpendicularly applied at the ends of equal arms. In the steelyard, the Power and the Weight are perpendicularly applied at the ends of unequal arms. Pincers, scissors, and snuffers, are double levers of this kind, the rivet being the fulcrum.

Since *CM* may be either greater or less than *CN*, the Power in levers of this class may be either less or greater than the Weight, and consequently "Force" may be either "gained" or "lost" by using them.

Of the SECOND class, a cutting blade, such as those used by coopers, moveable round one end, which is fastened by a staple to a block, by means of a handle fixed at the other end, is an example. An oar is also such a lever; the Fulcrum being the extremity of the blade (which remains fixed, or nearly so, during the stroke), the muscular strength and weight of the rower being the Power, and the Weight being the resistance of the water to the motion of the boat, which is counteracted and overcome at the rowlocks. A pair of nutcrackers is a double lever of the second class.

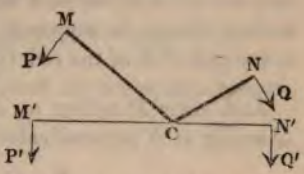
Here, since *CM* is greater than *CN*, the Power is always less than the Weight, or "Force is gained" by using levers of the *second* class.

An example of the THIRD class is the board which the turner (or knifegrinder) presses with his foot to put the wheel of his lathe in motion; the Fulcrum being the end of the board which rests on the ground, the Power being the pressure of the foot, and the Weight being the pressure produced at the crank put on the axletree of the wheel. Fire-tongs and sugar-tongs are double levers of this kind; the Weight in either case being the resistance of the substance grasped. The limbs of animals are also such levers: thus, if a weight be held in the hand and the arm be raised round the elbow as a Fulcrum, the weight is supported by muscles fastened at one extremity to the upper arm, and again attached to the fore-arm, after passing through a kind of loop at the elbow.

Here, since *CM* is less than *CN*, the Power is greater than *the Weight*, or "Force is lost" by making use of levers of the *third class*.

24. PROP. V. *If two forces acting perpendicularly at the extremities of the (straight) arms of any lever, balance each other, they are inversely as the arms.*

Let the forces P and Q, acting perpendicularly at the extremities of the straight arms CM and CN of any lever whose fulcrum is C, balance each other; they are inversely as the arms.



Draw the horizontal line $M'N'$ through C, and take $CM'=CM$, and $CN'=CN$. Let a force P', equal to P, act perpendicularly at M'; and a force Q', equal to Q, act perpendicularly at N'.

Now since $CM'=CM$, and $P'=P$, P' will produce the same effect as P to turn the lever round; Axiom III. Art. 18;

But, by supposition, P balances Q;

$\therefore$ P' would balance Q.

And since $CN'=CN$, and $Q'=Q$, Q' will produce the same effect as Q to turn the lever round; Axiom III.

But Q would balance P';

$\therefore$ Q' would balance P';

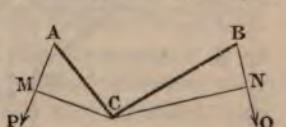
wherefore, by Prop. II., $P' : Q' :: CN' : CM'$,

and $\therefore$ $P : Q :: CN : CM$.

Wherefore, "If two forces, &c." Q. E. D.

25. PROP. VI. *If two forces acting at any angles on the arms of any lever, balance each other, they are inversely as the perpendiculars drawn from the fulcrum on the directions* (i. e. *the lines*) *in which the forces act.*

Let P and Q be two forces which acting at any angles on the arms CA and CB (which may be either straight or crooked) of any lever ACB balance each other about the fulcrum C; They are inversely as the perpendiculars CM and CN drawn from the fulcrum on the lines in which they act.



Since a force produces the same effect at whatever point in its line of action it is applied (Art. 15), the force P may be supposed to be applied at M; and in order that it *may* be so applied, let a rod (CMA) without weight be fastened to CA. In like manner Q may be supposed to be applied at N perpendicularly to the part CN of the rod CNB which is added to CB.

And, since P acting at M perpendicularly to CM balances Q acting at N perpendicularly to CN; $\therefore$, by Prop. V.,

$$P : Q :: CN : CM;$$

and therefore when P and Q acting at A and B in the lines AMP, BNQ balance, they are as $CN : CM$.

Wherefore, "If two forces, &c." Q. E. D.

26. [Cor. Conversely. If P and Q are inversely as the perpendiculars from the fulcrum upon their lines of action, they will balance each other.

For suppose Q' to be the force which applied at B in BN balances P at A.

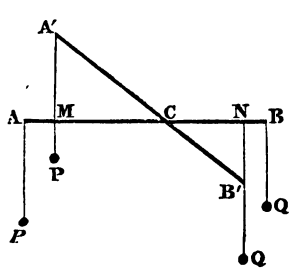
Then, by the proposition, $P : Q' :: CN : CM$;

But, by supposition, $P : Q' :: CN : CM$;

$\therefore P : Q' :: P : Q$, whence $Q' = Q$;

but Q' balances P, $\therefore$ Q also balances P.]

27. Prop. VII. *If two weights balance one another on a straight lever when horizontal, they will balance each other in every position of the lever.*



Let P and Q be two weights that balance each other round the fulcrum C on the lever ACB when it is horizontal; They will balance each other on the lever when it comes into any other position, as $A'CB'$.

Produce QB' to cut AB in N.

Since weights act perpendicularly to the horizon, $A'P$ and $NB'Q$ are each perpendicular to the horizontal line ACB;

$\therefore$ angle $CMA' =$ right angle $=$ angle CNB',

and angle $A'CM =$ opposite angle $B'CN$; Euc. I. 15.

$\therefore$ also angle $CA'M =$ angle $CB'N$, and the triangles are equiangular, and therefore similar.

Therefore, by Euc. VI. 4,

$$CN : CB' :: CM : CA';$$

$$\therefore CN : CM :: CB' : CA' :: CB : CA.$$

But, $\because$ P and Q balance on ACB, $\therefore$ $CB : CA :: P : Q$;

$$\therefore CN : CM :: P : Q;$$

But CM and CN are the perpendiculars from C on the lines in which P and Q act, when they are hung at A' and B'; therefore, by Prop. VI., Cor., P and Q will balance on $A'CB'$.

Wherefore, "If two weights, &c." Q. E. D.

CHAPTER III.

THE COMPOSITION AND RESOLUTION OF FORCES.

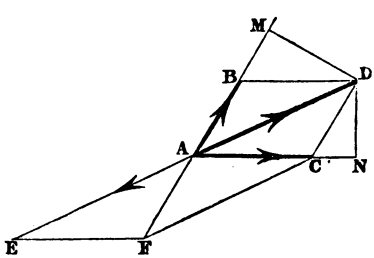
28. Definition of Component and Resultant Forces.

It is found, by experiment, that a body which is acted on by two forces applied at the same instant and in different lines to the same point of it, instead of having a tendency to move in either of the lines the forces act in, has a tendency to move in a line lying between them. Whence it appears, that the two original pressures by their combined action produce a third pressure; which pressure is called, from the circumstance of its "resulting" out of the actions of the original pressures, their "Resultant" with respect to them; while they are called, with *respect to it, its "Components."*

The Resultant (R), which produces the same effect as the compound action of the original forces P and Q applied at the same point at the same instant, is said to be "compounded" of P and Q. This Resultant (R) also, if conceived to be the sole original force, may be supposed to be "resolved" into the two forces P and Q; since those two forces, acting in the manner described (namely, at the same point, and at the same instant), produce exactly the same effect on the body as the single force R does.

[An instance of the "Composition of Forces" is a boat moored in a stream by ropes attached to the boat and to either bank; the Component forces here being the tensions of the ropes, and their Resultant being the force by which the pressure of the stream on the boat is counteracted.]

29. PROP. VIII. *If the adjacent sides of a parallelogram represent the component forces* [*that act at a point at the same instant*], *in direction* [*i.e. in their lines of action*] *and magnitude, the diagonal will represent the resultant force* (1) *in direction* [*line of action*], *and* (2) *in magnitude.*



Let AB and AC represent, in direction and magnitude, the two component forces that act at A. Complete the parallelogram BC, and draw the diagonal AD. Then AD will represent the resultant of AB and AC, (1) in direction, and (2) in magnitude.

From D draw DM and DN perpendicular to AB and AC, produced if necessary.

(1) Then in the triangles DBM, DCN,

angle DMB = right angle = angle DNC,

and angle DBM = angle BAC (since BD, AC are parallel *and* MBA *cuts* them), = angle DCN;

$\therefore$ the third angle, BDM, of the one triangle = the third angle, CDN, of the other; and the triangles are equi-angular and similar.

$$\therefore CD : DN :: BD : DM,$$

and $\therefore$, alternately, $CD : BD :: DN : DM$.

Now if there be a lever AD whose fulcrum is D, which is acted on by the forces AB, AC applied at A; since

Force in the line AM: force in the line $AN :: AB : BC$

$:: CD : BD$, since BC is a parallelogram,

$:: DN : DM$,

the two forces acting on the lever AD are inversely as the perpendiculars from the fulcrum on their lines of action, and therefore the lever will be kept at rest about D by them (Prop. VI.). Wherefore the lever will also be kept at rest by the Resultant of those forces; because that single force produces the same effect as they do when they act at the same point and at the same instant.

This Resultant therefore *must act in the line* AD, for it keeps the lever at rest, which it could not do were it to act at A and make any angle with the lever AD on either side of it.

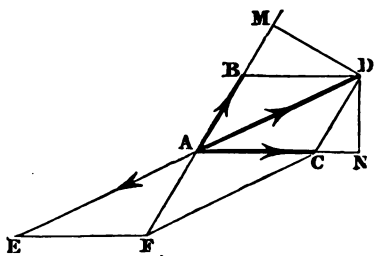
(2) Again: Having shown that the resultant of AB and AC acts in the line of the diagonal, next to prove that the diagonal represents it in *magnitude* as well as in direction.

Produce DA, and suppose a force AE to be taken in it equal and opposite to the Resultant of AB and AC.

The joint effect of AB and AC will now be counteracted by AE; and the point A, which is acted on by the three forces AB, AC, and AE, will remain at rest.

Whatever, therefore, be the effect produced by the joint action of AE and AC, it is counteracted by AB; that is, AB must be *equal and opposite* to the Resultant of AE and AC.

Complete the parallelogram EC, and draw the diagonal AF.



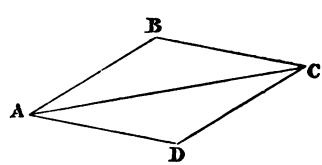
By the first part of the Proposition, AF is the line in which the resultant of AE and AC acts; and since the force AB is equal and opposite to that resultant, AF must be in the same straight line with AB, and, therefore, it is parallel to CD.

Wherefore FD is a parallelogram; and EC and FD being parallelograms, $AE = FC = AD$.

The Resultant, therefore, of AB and AC (which is equal and opposite to AE) will be properly represented in *magnitude* by AD, the diagonal of the parallelogram of which AB and AC are the sides.

Wherefore, "If the adjacent sides, &c." Q. E. D.

30. PROP. IX. *If three forces represented in magnitude and direction by the three sides of a triangle [when taken in order], act on a point, they will keep it at rest.*



Let the sides AB, BC, CA, taken in order,* of the triangle ABC, represent in magnitude and direction (see Art. 15) three forces that act on the point A; They will keep A at rest.

Complete the parallelogram BD.

Then AD is parallel and equal to BC; and it will, therefore, represent in magnitude and in *line of action* (Art. 15) the force which acts at the point A in the *direction* BC and is represented in magnitude by BC.

* By the expression "taken in order" it is meant that if ABC be the triangle, and AB be one of the forces, BC (and not CB) is the next, and CA (*not* AC) is the third; so that the forces are described in the same direction round the triangle, proceeding from A to B, from B to C, and from C to A again.

Now the forces AB and AD acting at A will produce a resultant AC. Prop. VIII.

If, therefore, a force CA act at A, the force AC will be counteracted, and the point A will remain at rest. Wherefore, if three forces represented in magnitude and direction by AB, AD, CA,—(or, which is the same thing, if they be represented by the three sides, AB, BC, CA, taken in order, of the triangle ABC)—act on a point A, they will keep it at rest. Q. E. D.

31. [Cor. 1. It appears from this proof, that if two sides of a triangle ABC, as AB and BC, taken in order, represent in magnitude and direction two forces which act at the same instant at a point A, the third side of the triangle, AC, not taken in the same order as AB and BC, represents their Resultant in magnitude and direction.]

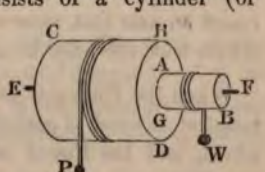
32. [Cor. 2. Since the forces which keep the point at rest are represented by the sides of a triangle, it follows that the sum of any two of them must necessarily be greater than the third. Eucl. I. xx.]

CHAPTER IV.

MECHANICAL POWERS.

33. The Mechanical Powers are certain instruments by which a lesser heavy body (or other force), called the *Power*, may be made to balance a greater, called the *Weight*. The properties will now be investigated of three of them, called the *Wheel and Axle*, the *Pulley*, and the *Inclined Plane*. They will be supposed to be rigid bodies and without weight.*

34. The Wheel and Axle consists of a cylinder (or *Axle*) AB, terminating in another cylinder (or *Wheel*) CD of greater base. The two cylinders have a common axis, EF, which is supported in a horizontal position. The *Power* (P) is a heavy body which hangs freely by

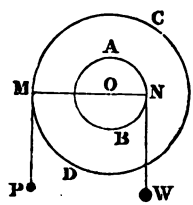


* The other Mechanical Powers are the Lever, the Toothed Wheel, the Wedge, and the Screw.

a string coiled round the *Wheel;* and the *Weight* (*W*) is another body, hanging by a string coiled about the *Axle*, and tending to turn the machine round in the opposite direction.

35. PROP. X. *There is an equilibrium on the Wheel and Axle when the Power is to the Weight as the radius of the Axle is to the radius of the Wheel.*

Suppose the strings by which the Power and the Weight hang to be in the planes of the circles, which are those ends of the two cylinders that lie in the same plane.* Since the cylinders have a common axis, these circular ends have a common center; let *O* be it, and join *O* with *M* and *N*, the points in which the strings leave (and therefore touch) the circumferences of the circles. *OM* and *ON* are therefore perpendicular to *MP* and *NW*, the lines in which the Power and the Weight act.



Now *P* and *W* act perpendicularly to the lines *OM* and *ON*, and balance each other; and therefore, supposing *MON* to be a lever kept at rest round *O* as fulcrum,

P : *W* :: *ON* : *OM* :: radius of axle : radius of wheel.

When, therefore, this ratio of the Power to the Weight is fulfilled on the Wheel and Axle, there will be equilibrium. Q. E. D.

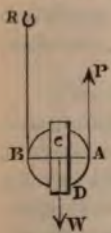
36. [COR. Since *OM* and *ON* are both perpendicular to the vertical lines in which *P* and *W* act, and also pass through the same point *O*, *MON* is a horizontal straight line.]

37. DEF. THE PULLEY consists of a *grooved wheel* moveable round an axis that passes through the center of the wheel, and whose ends are fixed in a frame, called the *Block*.

When the Pulley is used as a Mechanical Power, a string fastened at one end is passed round part of the circumference of the wheel, and the Power is applied at the other end of the string. The Weight is attached to the Block.

* *These* circles are *HD* and *AG* in the figure to Art. 34.

PROP. XI. *In the single moveable pulley where the strings are parallel, there is an equilibrium when the Power is to the Weight as* 1 *to* 2.



Let PA, which touches the pulley at A, be the line in which the Power (P) acts. Join A with C, the center of the pulley. Produce AC to B. Draw BR touching the circle; it is perpendicular to AB and parallel to AP; this, therefore, will be the other part of the string. Let W, the Weight, act at C by means of the block, in the line CW.

The whole being at rest, the string $RBDAP$ must be equally stretched throughout, or motion would ensue. The force upwards, therefore, exerted at the fixed point R is P; for it is the same as that which keeps AP stretched. Art. 14.

Now considering ACB as a straight lever kept at rest round C by forces P and P acting perpendicularly upwards at A and B, the pressure upwards on C will be $2\ P$ (Prop. II.); wherefore the Weight (W), which acts downwards at C and is supported by this pressure, is $= 2\ P$;

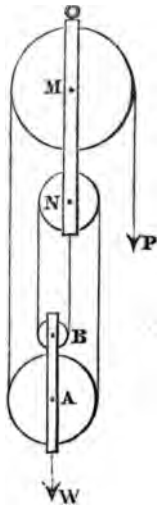
$$\text{or } P : W :: 1 : 2.$$

Wherefore, "In the single moveable pulley, &c." Q. E. D.

38. [COR. The pressure produced at C by the forces P and P acting at A and B is perpendicular to AB, or parallel to AP and BR. (Art. 18. Ax. I.); wherefore the line of action of the Weight is perpendicular to AB, or parallel to AP and BR. And therefore since in the case of equilibrium the strings are parallel to the line in which the weight acts, if the Weight be a heavy body hanging freely from C, AP and BR are vertical.]

39. PROP. XII. *In a system in which the same string passes round any number of pulleys, and the parts of it between the pulleys are parallel, there is an equilibrium when Power* (P) : *Weight* (W) :: 1 : *number of strings at the lower block.*

[In this Proposition, as in Prop. XI., the Power and the Weight act in a direction *parallel to* that of the strings.]



Because there is an equilibrium, the portions of the string which lie on either side of any pulley must be pulled by equal forces; and since the outside portion, to which the power is applied, is acted on by a force P, each portion is, therefore, pulled by a force P.

Wherefore the part of the weight that is sustained at A is (Prop. XI.) $2P$; and the same is true with respect to the pulley B; and so on for every other pulley at the lower block. If, therefore, there be n pulleys at the lower block, W, the whole weight supported, will be $n \times 2P$; or $W = 2nP$;

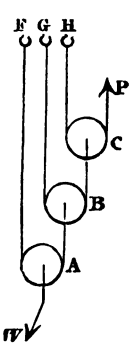
$$\therefore \frac{P}{W} = \frac{1}{2n},$$

i. e. $P : W :: 1 : 2n :: 1 :$ number of strings at the lower block.

Wherefore, "In a system, &c." Q. E. D.

40. PROP. XIII. *In a system where each pulley hangs by a separate string and the strings are parallel, there is an equilibrium when* $P : W :: 1 : 2$ *raised to that power whose index is the number of moveable pulleys.*

[The weight W is supposed to act in a line parallel to the strings.]



In the figure, C being a moveable pulley with parallel strings, there is equilibrium if the pressure downwards at $C = 2P$.

$\therefore$ Tension of $BC = 2P$;

$\therefore$ weight supported at moveable pulley $B = 2 \times (2P) = 2^2 P =$ tension of AB;

So, weight supported at moveable pulley $A = 2 \times (2^2 P) = 2^3 P$.

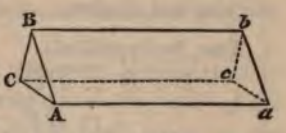
Wherefore, if W, "the Weight," be $= 2^3 P$, there is equilibrium on the system of *three* moveable pulleys here represented.

And if n were the number of moveable pulleys, it would appear, by the same mode of reasoning, that there would be equilibrium when $W = 2^n P$;

$$\therefore \frac{P}{W} = \frac{1}{2^n}; \quad \text{or, } P : W :: 1 : 2^n.$$

Wherefore, "In a system where each pulley hangs by a separate string, &c." Q. E. D.

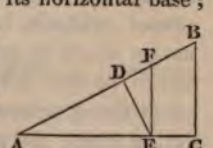
41. DEF. If a prism whose ends are similar and equal right-angled triangles (C and c being the right angles), and whose sides are rectangles, be placed so that either of the sides of the prism adjacent to the right angles of the ends (as $ACca$) be horizontal, the slant side ($ABba$) is called an *Inclined Plane.*



When the Inclined Plane is used as a Mechanical Power, the Power and the Weight both act in a plane parallel to either end of the prism, and the Inclined Plane is represented by a triangle such as ABC, of which the side AB is called the *Length* of the plane, AC its *Base*, and CB its *Height.* And the plane, by being supposed to be perfectly rigid and inflexible, will be capable of counteracting, and entirely destroying, the effect of any force which acts upon it perpendicularly.

42. PROP. XIV. *The weight (W) being on an Inclined Plane and the force (P) acting parallel to the plane, there is an equilibrium when $P : W ::$ the height of the plane : its length.*

Let AB be the length of the plane; AC its horizontal base; and BC, perpendicular to AC, its height. Let the Weight (W) act vertically at D, and the Power (P) act in the line DB which is parallel to the plane; Then, if $P : W :: BC : AB$, there is equilibrium.



From D draw DE at right angles to AB, meeting the base

in E; and from E draw EF vertical, or at right angles to AE, meeting AB in F.

Then in the triangles EFD and ABC,

$\because$ FE, BC are parallel, and AFB cuts them,

$\therefore$ angle DFE = angle ABC;

and angle FDE = right angle = angle BCA,

$\therefore$ angle DEF = angle CAB, and the triangles are equiangular and similar.

Now, by hypothesis,

$$P : W :: BC : AB$$
$$:: DF : FE, \text{ by equiangular triangles.}$$

But the same effect that is produced on D by the two forces which are represented in magnitude and direction by DF and FE, will be produced by their resultant. And their resultant DE (Art. 31, Cor.), being perpendicular to the plane AB, will be entirely counteracted and destroyed by that plane, and no motion, therefore, can be communicated by it to the body at D. The body at D, therefore, will remain at rest on the inclined plane when acted on by the two forces P and W which are such that $P : W :: BC : AB$.

Wherefore, "The Weight, &c." Q. E. D.

43. Def. By the Velocity of a body in motion is meant the Degree of Quickness, or Speed, with which the body is moving.

[This "degree of quickness" is described, or *measured*, by stating how long the line is that the body moves through, with unaltered swiftness, in some *given* portion of time. Thus a clear notion would be conveyed of the Velocity of a coach, if it were said to be *nine miles* in *an hour;* the *space* moved over by the coach being *nine miles*, and *an hour* being the *portion of time* during which the motion took place. If only the space that is moved through were mentioned, and nothing were said about the *time of describing* that space,—or if the time only were given,—

it is evident that no idea could be formed of *the degree of quickness*, that is, of the *Velocity*, of the coach's motion.

The Velocity of a body is *measured*, in mathematical investigations, by the number of *feet* passed over by the body in a *second* of time.

Cor. Since the quicker a body moves the more space it will pass over in a *given* time, it will follow from the observations just made, that *The Velocities of two bodies that move during any* (THE SAME) *time, are as the spaces which the bodies respectively describe—each with uniform swiftness—in that time.*]

44. Prop. XV. *Assuming that the arcs which subtend equal angles at the centers of two circles are as the radii of the circles, to show that if P and W balance each other on the Wheel and Axle, and the whole be put in motion, P : W :: W's velocity : P's velocity.*

Suppose P and W to act (as in Prop. x.) at the circumferences of the Wheel and the Axle, in the same plane, perpendicularly to the radii OM and ON; and let the whole be put into motion round the axis so that mOn may become the horizontal diameter. P will now act at m at right angles to Om, and W will act at n at right angles to On; and the velocities of P and W will be as Mm to Nn; since the former of these arcs is the length through which P will have descended in the same time that W has ascended through Nn.

$\therefore$ Vel. of P : vel. of W :: Mm : Nn, (Art. 43, Cor.),

:: OM : ON [by the assumption made in the enunciation],

:: W : P, by Prop. x.; because, by the hypothesis, P and W originally balanced each other.

Wherefore, "Assuming, &c." Q. E. D.

45. PROP. XVI. *To show that if P and W balance each other on the Machines described in Propositions XI., XII., XIII., and XIV., and the whole be put in motion,* $P : W ::$ *W's velocity in the direction of gravity : P's velocity* [in the direction in which it acts.]

(1) Let C, the center of the moveable pulley (see Fig. Prop. XI.), be raised through any height, as an inch; W will thereby be also raised through an inch, and each of the strings RB and AP will have been shortened an inch; so that, if P continue to keep the string tight, it will have moved through two inches in the time that W has been raised one inch.

Now $P : W :: 1 : 2$, Prop. XI.,

$::$ 1 inch : 2 inches,

$::$ W's velocity : P's velocity; Art. 43. Cor. Q. E. D.

(2) In the system where the same string passes round all the pulleys, and the parts of it between the pulleys are parallel, as in Prop. XII. (see Fig.), if the lower block be raised an inch, each of the strings between the upper and lower pulleys will be shortened an inch, and, there being $2n$ of such strings, P must have moved through $2n$ inches in the time that W moved through one inch, in order to have kept the string tight.

And $P : W :: 1 : 2n$; Prop. XII.,

$::$ 1 inch : $2n$ inches,

$::$ W's velocity : P's velocity; Art. 43. Cor. Q. E. D.

(3) In the system where each pulley hangs by a separate string and the strings are parallel (Prop. XIII.), if W be raised through an inch, and P have also moved through such a space that the strings are kept tight, A will have been raised through one inch, and B through two inches (by the first case proved in this Proposition). And B having been raised through two inches, C (by the first case) will have moved through 2×2, or 2^2, inches; and the next moveable pulley (the fourth) will have been raised *through* 2×2^2, or 2^3, inches. By the same reasoning, if n were *the number of moveable* pulleys, the highest of them will have

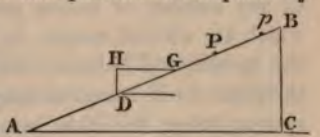
moved through 2^{n-1} inches, and the end therefore of the string by which P acts through 2^n inches; but

$P : W :: 1 : 2^n$, Prop. XIII.;

$:: 1$ inch $: 2^n$ inches

$::$ W's velocity : P's velocity; Art. 43. Cor. Q. E. D.

(4) Let the weight (W) be kept at rest at D on the inclined plane AB by the power (P) which acts parallel to the plane by means of a string DP. Suppose P, by moving through the space Pp, raises W through an equal space DG. Through G draw GH horizontal, and through D draw DH vertical. Then, by being moved through DG, W has been raised through a *vertical* height DH,—DH being the vertical line which cuts the horizontal lines passing through the two positions of W. Since, therefore, in the time that P moves in the line of its action through a space equal to DG, the *vertical* height that W moves through is DH, DG is to DH as the velocity of P in the direction of its action is to the velocity of W in the direction of gravity.



Now, in the triangles GDH and ABC,

since GH is parallel to AC, and AG meets them;

$\therefore$ angle $DGH =$ alternate angle BAC;

and since DH, being vertical, is parallel to BC, and DB meets them;

$\therefore$ angle $GDH =$ alternate angle ABC;

$\therefore$ also angle $DHG =$ angle BCA, and the triangles are equiangular, and $\therefore$ similar.

And $P : W :: BC : AB$; Prop. XIV.;

$:: DH : DG$, by similar triangles.

$::$ W's vel. in direction of gravity : P's vel. in the direction of its action; Art. 43, Cor.

Wherefore "If P and W balance, &c." Q. E. D.

CHAPTER V.

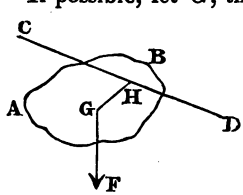
THE CENTER OF GRAVITY.

46. DEF. THE CENTER OF GRAVITY of a body, or of a system of any number of bodies connected together by inflexible bars which are without weight, is that point upon which the body, or the system, will balance when placed in any position whatever; the point itself being maintained at rest, and the body, or the system, being moveable in any way about the point, and being acted on by no other pressure than that arising from the weight of the matter composing it.

[The Center of Gravity of a body is, as it were, that *fulcrum*, round which the body, when placed in any position, has, of itself, no sort of tendency to turn, although the body be *capable* of being moved in any way about that fulcrum.]

47. PROP. XVII. *If a body balance itself upon a line in all positions, the center of gravity of the body is in that line.**

If possible, let G, the center of gravity of the body AB, *not* lie in the line CD on which the body balances in all positions; and let the body and line be so placed, that G and CD may be in the same horizontal plane. From G draw GH perpendicular to CD, and GF vertical.



Now since (by Def. Art. 46) if G be supported the body remains at rest when turned round the line into *any* position, the vertical pressure, therefore, which arises from the weight of the matter composing the body, must *always* pass through G;—or else

* That is to say—If there be a line round which a body can be made to revolve, such that if when the line is fixed in *any* position, the body, after being made to revolve round the line into *any* position, remains at rest without having any tendency to move, the center of gravity of the body lies in that line.

that pressure could not be rendered inoperative (as it is supposed to be) by G being maintained at rest. Wherefore, in the position of the body represented in the figure, this pressure will act perpendicularly at an arm GH, and have a tendency to turn the body round the point H, which (since H is a point in a line CD fixed in position) may be considered to be a fixed fulcrum. But, by hypothesis, when CD is supported the body balances in *all* positions upon it. Wherefore G does not lie out of the line CD, *i.e.* it lies in it. Q. E. D.

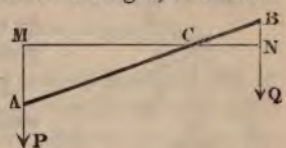
48. PROP. XVIII. *To find the center of gravity of two heavy points,* and to show that the pressure at the center of gravity is equal to the sum of the weights in all positions.*

Let P and Q be the weights of two heavy points A and B. Join AB by a straight inflexible bar without weight, and take

$$AC : AB :: Q : P + Q;$$

and $\therefore AC : AB - AC$, or CB,

$$:: Q : (P + Q) - Q, \text{ or } P.$$



Let ACB be in any position. Through C draw MCN horizontal; and through A and B draw the vertical lines MAP and BNQ; these last are the lines in which the weights P and Q act.

Then, the angles at M and N being right angles, and the angle ACM being equal to the opposite angle BCN, the angle CAM is equal to the angle CBN, and the triangles ACM and BCN are equiangular, and $\therefore$ similar.

Now, supposing P to act on the horizontal lever MCN at M at right angles to CM (Art. 13), and Q to act at N at right angles to CN, since

$$P : Q :: CB : CA$$

$$:: CN : CM, \text{ by similar triangles,}$$

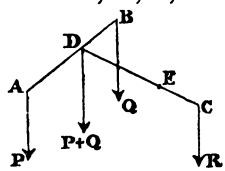
* The "heavy points" spoken of in this Proposition and the next are exceedingly small material bodies, and not geometrical points. For a geometrical point (being defined by Euclid to be "without parts") does not possess length, or breadth, or thickness, and consequently can be of no weight, since it can contain no matter.

therefore P and Q will balance on a fulcrum placed at C, and the pressure on C is $P+Q$, acting vertically. Props. II. and VII.

And this being true for *any* position, is true for *all* positions of A and B. Wherefore (by the definition of the Center of Gravity), C is the Center of Gravity of A and B; and the pressure at C, in all positions of A and B, is $P+Q$, acting vertically downwards.

49. PROP. XIX. *To find the center of gravity of any number of heavy points; and to show that the pressure on the center of gravity is equal to the sum of the weights in all positions.*

Let A, B, C, three heavy points whose weights are P, Q, R, be connected together and placed in any position.



Join AB, and take D a point in AB such that

$$AD : AB :: Q : P+Q;$$

$$\therefore AD : AB-AD, \text{ or } DB, :: Q : (P+Q)-Q, \text{ or } P;$$

therefore D is the center of gravity of A and B, and the pressure produced by P and Q in all positions of the system, is a pressure $P+Q$ acting vertically at D. Prop. XVIII.

Join DC, and take in DC a point E such that

$$DE : DC :: R : P+Q+R;$$

$$\therefore DE : EC :: R : P+Q;$$

therefore E is the center of gravity of the weight $(P+Q)$ acting at D and R acting at C; and if E be supported, those weights are supported in any position of the system. Since therefore the system will balance itself in all positions on E, that point is its Center of Gravity;—and the Pressure on E is $P+Q+R$.

The construction here applied to a system of three bodies may *be extended to* a system of any number of bodies.

Wherefore "The center of gravity of any number of heavy points may always be found, and the pressure on the center of gravity is equal to the sum of the weights." Q. E. D.

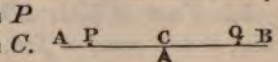
50. [By the definition given in Art. 46 of "the Center of Gravity" of a body, it will be understood that to have a center of gravity a body must have *Weight.* Now in the next two Propositions it is required to find the centers of gravity of a *line*, and of a *plane;* the former of which is defined by Euclid to have *length* merely, without either *breadth* or *thickness;* and the latter, though possessing *length* and *breadth*, is defined to be without *thickness.* A geometrical line, or plane, therefore, *can* have no weight; since there can be no weight where *matter* does not exist, and when *matter* exists under any form whatever, it is of *three* dimensions, or has *length*, and *breadth*, and *thickness.* The line, therefore, and the plane, of which it is required to find the centers of gravity, are not the line and plane of geometry.

But the line of which the Center of Gravity is determined in the next Proposition is supposed to be formed of very small equal heavy bodies placed, at the same distance one from the other, along the whole length of the line. And the plane triangle referred to in the next Proposition but one, is supposed to be made up of such lines, arranged parallel to any one of the sides of the triangle, and at equal distances one from the other.]

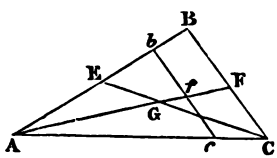
51. PROP. XX. *To find the center of gravity of a straight line.*

Let AB be a straight line composed of small equal heavy bodies ranged at equal distances one from the other from end to end of it.

Bisect AB in C, and let P and Q be two of the small heavy bodies equally distant from C. Then P and Q will balance in every position on C. And since the same is true of all such pairs of heavy bodies that are equidistant from C, the whole line will balance on C in every position, and therefore C is the center of gravity of the line.



52. PROP. XXI. *To find the center of gravity of a triangle.*



Let ABC be a triangle formed of lines ranged, at equal distances, parallel to any one of the sides, the lines themselves being made up of small equal heavy bodies placed at equal distances. Let bfc, parallel to BFC, be one of these lines.

Bisect AB in E and BC in F; join AF, CE by lines intersecting in G;—G is the center of gravity of the triangle.

For $Af : fb :: AF : FB$ (from the equiangular triangles Abf, ABF),

$:: AF : FC$, $\quad \because BF = FC$;

$:: Af : fc$ (from the equiangular triangles AFC, Afc).

And since the first and third terms of this proportion are the same, fb is equal to fc; and therefore the straight line bc would balance in any position on f. Prop. XX.

In the same manner all the other lines parallel to BC might be shown to balance in any position on the points in which they are cut by AF; therefore the whole triangle would balance on AF in any position; wherefore the center of gravity of the triangle is in AF.

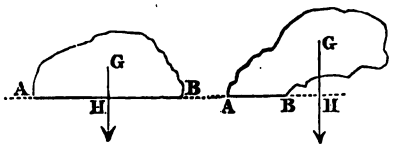
Similarly it may be shown that the center of gravity of the triangle is in the line CE.

Wherefore G, the intersection of AF and CE, is the center of gravity of the triangle ABC.

53. PROP. XXII. *When a body is placed on a horizontal base, it will stand or fall, according as the vertical line drawn from its center of gravity falls within or without the base.*

Let GH be a vertical line drawn through G the center of gravity of the body, and meeting the horizontal plane on

which the body is placed in the point H; and let ABH be any horizontal line whatever drawn through H and terminated at A and B, points in the boundary of the base of the body.



(1) Let H fall *within* the base; it is therefore *between* A and B.

Then AB being considered as a horizontal lever acted on at H (Art. 13) by the weight of the body, no motion can take place round A, because the tendency of the body's weight would be to draw the lever *downwards* round A, a motion which the resistance of the plane will prevent. For the same reason no motion can take place round B; and therefore the body will fall over neither at A nor B.

In the same manner it may be shown that the body will not fall over at either extremity of *any other* horizontal straight line which is drawn through H and is terminated by the boundary of the base.

Wherefore the body will remain at rest when the vertical drawn from its center of gravity falls *within* the base.

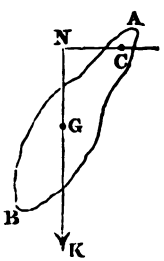
(2) If H fall *without* AB, as in the second figure, no motion of the lever ABH can take place round A as fulcrum, because the effect of the weight acting at H would be to draw AH *downwards*, a motion which the plane prevents. But if B be the fulcrum, the effect of the weight at H would be to make the line AB move *upwards* from the plane round B, and as there is no force to prevent this motion taking place, the body, though it were prevented from falling over at every other point of the boundary of the base, would still fall over round B. When, therefore, H falls outside the horizontal base, the body cannot stand, but must fall over at some point or other in the boundary of the base.*

Wherefore, "When a body, &c." Q. E. D.

* The demonstration of the second case requires it to be understood that no line whatever can be drawn through H meeting the circumference of the base in *points which* are on opposite sides of H.

54. PROP. XXIII. *When a body is suspended from a point* [round which it can swing freely] *it will rest with its center of gravity in the vertical line passing through the point of suspension.*

Let AB be the body in any position, C the point from which it is suspended, G its center of gravity, NGK a vertical line drawn through G, CN a perpendicular from C on NGK.



Since the weight of the body acts in the vertical line which passes through G the center of gravity, it may be supposed to be applied at N; and, CN being considered as a lever moveable round a fulcrum C, as there is no force to counteract W acting perpendicularly at N on CN, motion must ensue.

But if NG pass through C, CN vanishes, the weight is sustained by the immoveable fulcrum C, and the body is at rest.

Wherefore, " When a body, &c." Q. E. D.

THE ELEMENTS

OF

HYDROSTATICS.

The explanatory articles and paragraphs which are enclosed in brackets form no part of the Course adopted by the University. Those in smaller type are illustrative of the Definitions and Propositions they immediately follow. The whole of the extra matter so introduced forms but a very small and a very easy portion of the book, and will be found useful in solving Problems.

CHAPTER I.

55. THE SCIENCE of HYDROSTATICS investigates the conditions fulfilled by the pressures applied to, or produced by, fluids that are in a state of rest.

56. DEF. A FLUID is a material body of which the parts can be divided in any direction and can be moved among one another by any force, however small.

[Mercury, Water, Air, Gas, Steam, are all instances of fluids.]

57. FLUIDS have been divided into *Elastic*, and *Non-elastic*.

(1) ELASTIC FLUIDS are those of which the dimensions are increased or diminished when the pressure upon them is diminished or increased.

[Air is an elastic fluid, as will hereafter be proved. So also is Gas, and Steam.]

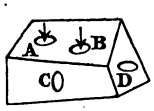
(2) NON-ELASTIC FLUIDS are those of which the dimensions are not diminished by the addition of pressure or increased by the withdrawal of pressure.

[Water, Mercury, and probably all other liquids, are compressible, but in a slight degree only. They offer, however, so much resistance to compression, that the conclusions obtained on the supposition of their being entirely incompressible are free from any sensible error, except in cases where the pressure is exceedingly great.]

CHAPTER II.

PRESSURE OF NON-ELASTIC FLUIDS.

58. PROP. I. *Fluids press equally in all directions.*



Let a close vessel of any shape be filled with fluid, and let *A*, *B*, *C*, and *D*, *similar* and *equal* portions (of any size and shape) of the sides of the vessel, be removed, and their places supplied by plugs fitting the orifices exactly and acted upon by pressures just sufficient to keep them at rest. Then if an additional force *P* be applied to any one of the plugs, it is found, by experiment, that an *equal* additional pressure must be applied to *every other* of the plugs to prevent the fluid from bursting out.

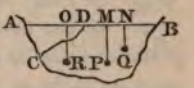
This proves that a pressure communicated to the surface of a fluid at rest is transmitted by the fluid, undiminished and unimpaired, to every other equal and similar portion of the surface by which the fluid is bounded.

Also, whatever be the *directions* in which any of these plugs are inserted through the sides of the vessel, the same quantity of pressure is required to keep each of them at rest; which shows that, at the same point in a fluid, the fluid presses with *equal* force in *all* directions.

Wherefore, "Fluids press equally in all directions." Q. E. D.

59. PROP. II. *The pressure upon any particle of a fluid of uniform density is proportional to its depth below the surface of the fluid.*

Let the vertical lines PM and QN be drawn through P and Q, two equal particles situated in the interior of an uniformly dense fluid which is at rest, and let them meet the surface of the fluid in the points M and N.



Suppose the pressures on P and Q to be produced by the weight of fluid particles, of the same magnitude as P and Q, placed, touching each other, along the lines PM and QN. Then, since the fluid is of uniform density, the weights of such lines of particles will be proportional to their lengths.

$\therefore$ Pressure on the particle P

: pressure on the particle Q

:: weight of the line PM of particles

: weight of the line QN of particles

:: PM : QN.

Again; if a piston be inserted through the side of an open vessel containing fluid, so as to test the amount of the pressure at any point R, it is found by experiment that whether a line of fluid particles extends vertically to the surface, or a portion of such line is cut off by the side of the vessel intervening, the pressure at R is the same.

Wherefore, "The pressure, &c." Q. E. D.

60. PROP. III. *When a fluid is at rest its surface is horizontal.*

[The manner in which the matter composing non-elastic fluids acts is two-fold; first, by the particles pressing with their weights on those immediately below them; and, second, by the property these particles possess of communicating, in all directions and without diminution, any pressure to which they are subjected.]

Let there be two contiguous particles, the centers of which are the points P and Q, situated in the same horizontal plane in a fluid at rest. Since the action of gravity on these particles is *perpendicular* to the horizontal plane in which they lie, its action on either of them can produce no effect *in* that plane. It is in consequence, therefore, of the action of the fluid which surrounds them, that the particles have no horizontal motion: and this action must be the same on each, or motion would ensue.

But since fluids press equally in *all* directions (Prop. I.), and the *horizontal* pressures on the particles cannot but be equal, the vertical pressures on them, which are as the distances Pa and Qb below the surface of the fluid (Prop. II.), must also be equal; and therefore

$$Pa = Qb;$$

$\therefore$ ab is parallel to PQ, and is $\therefore$ horizontal.

And since, in like manner, the line joining *any two* adjacent particles in the surface of the fluid may be shown to be horizontal, the surface itself is horizontal.

Wherefore, "When a fluid, &c." Q. E. D.

61. PROP. IV. *If a vessel, the bottom of which is horizontal and the sides vertical, be filled with fluid, the pressure on the bottom will be equal to the weight of the fluid.*

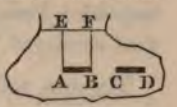
The sides of the vessel being vertical, and the whole of the fluid being conceived to be made up of vertical straight lines of fluid particles, each of these lines will press vertically with its weight, and the sum of these vertical pressures will be the weight of the fluid in the vessel. Now the base of the vessel, being horizontal, will counterbalance all the vertical pressures upon it, and destroy them entirely. The pressure, therefore, sustained by the horizontal base of a vessel whose sides are vertical, is equal to the weight of the fluid in the vessel. Q. E. D.

62. [From Propositions II. and IV. the following conclusion may be drawn.

The pressure of a fluid on any horizontal plane placed in it, is equal to the weight of a column of the fluid whose base

is the area of the plane, and whose height is the depth of the plane below the horizontal surface of the fluid.

Let AB and CD be two equal areas in the same horizontal plane. Since the pressure is the same on every point in the same horizontal plane, and (the areas AB and CD being equal) the number of particles in the plane CD is the same as that in the plane AB, therefore the pressure on AB = pressure on CD.



Let AB be such an area that a vertical line of fluid particles reaches from every particle in it to the surface; and suppose the rest of the fluid to become solid: $EABF$ then becomes a vessel with vertical sides and horizontal base, and the pressure on AB = weight of the column EB of fluid. Prop. IV.

Therefore, the pressure on any *horizontal* area CD = weight of a column of the fluid whose base is CD, and whose height is the depth of CD below the horizontal plane of the surface of the fluid in the vessel. Q. E. D.]

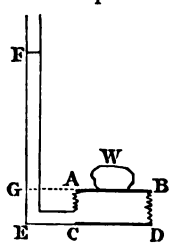
63. [From Art. 49 it appears that the pressure on the Center of Gravity of a system of bodies considered as heavy points, is the same as if the weights of the bodies were collected at it. So long therefore as the Quantity of Matter in the system remains the same, the amount of Pressure produced in the direction of gravity by the weights of the several parts of the system is the same whatever be the manner in which they may be arranged.

Now though at first sight it might appear probable that the pressure on the bottom or the sides of a vessel filled with fluid would depend (somehow or other) on the *quantity* of the fluid by which that pressure is produced, such is not the case; for it is found, both from experience and by theory, that the pressure produced by a fluid, at rest, on the surface of the vessel containing it, is not dependent on the quantity of the fluid; a fact apparently so much at variance with the law governing the amount of pressure produced by solid bodies, that it has been called THE HYDROSTATIC PARADOX.]

64. Prop. V. *To explain the Hydrostatic Paradox.*

The Hydrostatic Paradox is this, "Any Pressure, however small, may, by means of a fluid, be made to counterbalance *any other* Pressure, however large, whatever be the quantity of fluid employed."

The top and bottom of a vessel AD are boards which are connected together by leathern sides. The vessel communicates with a vertical tube EF of small uniform bore, by means of a horizontal pipe CE. Let (A) be the number of square inches in the area of the board AB, and (a) the square inches in the area of a horizontal section of the pipe EF.



Let AB be held in a horizontal position, and water be poured into the tube until it just rises in AD to AB. The water will therefore rise in the tube to G, a point in the same horizontal plane as AB. Prop. III.

If now there be a heavy weight of W lbs. laid upon AB, it is found that it can be supported at rest by a small quantity (FG) of water poured into the tube.

To show the reason of this;

Since the sides of the tube FG are vertical, the pressure on the horizontal section of the tube at G is the weight of the fluid column FG. Suppose w to be this weight, in pounds.

Then, since the pressures at every point in the same horizontal plane of a fluid at rest are the same,

$\therefore$ Pressure on the area (A) of AB : pressure on the area (a) at $G :: A : a$,

$$\text{i. e., } W : w :: A : a;$$

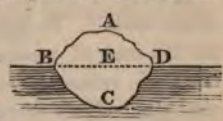
$$\therefore w = W \times \frac{a}{A}.$$

If therefore W and A be given, w may be made as small as may be wished by diminishing a; and the quantity of the fluid *employed* does not enter into this result, and therefore cannot *affect it.* Q. E. D.

[Whether the pressure on the area (φ) at G be produced by the weight of the fluid column GF or by means of a plug acted on by some force, the pressures on the areas (a) at G and (A) at AB will still bear to one another the ratio of a to A; a ratio which is wholly independent of the quantity of fluid contained in the vessel.]

65. PROP. VI. *If a body float on a fluid, it displaces as much of the fluid as is equal in weight to the weight of the body; and it presses downwards, and is pressed upwards, with a force equal to the weight of the fluid displaced.*

Let $ABCD$ be a body that displaces the portion $BEDCB$ of the fluid on which it floats.



The weight of the floating body produces a pressure which acts vertically downwards. Therefore the pressure of the fluid which keeps the body at rest must act vertically upwards, and be equal to the weight it balances.

Now suppose the floating body to be removed, and the space $BEDCB$ filled with fluid the same as that on which the body floated. The equilibrium of the fluid will not be disturbed, neither will the pressure of that part of it which was formerly in contact with the surface of the floating body be altered, if the particles of fluid in $BEDCB$ be supposed to become permanently connected with one another and to form a solid.

Let this take place; then the pressure downwards of the part $BEDCB$ of the fluid which becomes solid is its weight. And since this pressure is counteracted by the same sustaining power as that which balanced the weight of the floating body, the weight of the floating body must be equal to the weight of the fluid it displaces.

Wherefore, "If a body float on a fluid, &c." Q. E. D.

CHAPTER III.

SPECIFIC GRAVITIES.

66. THE BULK, or VOLUME, or CONTENT, or MAGNITUDE, of a solid body is measured by the number of times it contains that of some particular body previously fixed upon as a standard of magnitude.

A cube of which an edge is an inch in length is called "a cubic inch." In the following pages *a cubic inch* will be taken for the unit of measurement; so that when it is said that M is the *bulk*, *volume*, *content*, or *magnitude*, of a body, it is meant that the body is M cubic inches in size.

67. DEF. The SPECIFIC GRAVITY of any substance is the weight of an unit of its magnitude.

If, as stated in the last Article, the magnitude of a body be measured by the number of *cubic inches* it contains, and the weight of one cubic inch of the substance the body is composed of be given in *grains*, then it will follow, that, S being taken to represent the Specific Gravity of any substance, S is the *number* of *grains* that *one cubic inch* of that substance *weighs*.*

* The Tables which are termed "TABLES OF SPECIFIC GRAVITIES" give the Ratios which the weights of bulks of various substances bear to *equal* bulks of water. In other words, they give the number of times that the weight of any bulk of each of the substances contains the weight of an *equal* bulk of water.

It having been found by experiment that the weight of a piece of Iron : the weight of a bulk of Water *of the same size* :: 7·8 : 1, and that the weight of a piece of Silver : weight of an equal bulk of Water :: 10·5 : 1, and so for other substances—tables have been formed in which the numbers 1, 7·8, 10·5, &c., are placed opposite the words, "Water," "Iron," "Silver," &c. By means of these tables (as will be shown), the weight of any bulk of any of the substances so registered can be determined, if the weight be known of some particular bulk of any one of them.

[*The numbers given* in these Tables are generally called the "Specific

68. PROP. VII. *If M be the Magnitude of a body, S its Specific Gravity, and W its Weight, $W = MS$.*

Let M = number of cubic inches in the body.

S = Specific Gravity of the body; i. e. let S be the number of grains that one cubic inch of it weighs.

W = number of grains that the whole body weighs.

Then since $\underset{\text{grains.}}{W} : \underset{\text{grains.}}{S} :: \underset{\text{c. inches.}}{M} : \underset{\text{c. inch.}}{1}$;

$\therefore\ W = M \times S.$ Q. E. D.

Gravities" of the several substances registered; but the enunciation of Prop. VII. Art. 68, will not permit them being so called here.]

"*The Tables of Specific Gravities*" give—

Platina	21·5	Zinc	6·9
Gold	19·4	Sea Water	1·027
Mercury	13·6	Water	1·
Lead	11·4	Proof Spirit	0·930
Silver	10·5	Pure Alcohol	0·825
Copper	8·9	Air, at the surface of the Earth—the average	0·00125
Iron	7·8		
Tin	7·3		

Hence it may be shown that—

i. *The weights of any two substances that are of equal bulk are in the ratio of the numbers given by the Tables as corresponding to the substances.*

For w, w', w'' being the respective weights of equal bulks of Water and of any two substances, as Iron and Silver, since, as explained above,

$$w' : w :: 7{\cdot}8 : 1, \text{ and } w : w'' :: 1 : 10{\cdot}5;$$

therefore, compounding these proportions,

$$w' : w'' :: 7{\cdot}8 : 10{\cdot}5.$$

So that if it be required

ii. *To find the weight of a cubic foot of Iron, having given that the weight of 10 cubic inches of Silver is 61 ounces nearly—*

∵ Weight of a cubic foot (or $12 \times 12 \times 12$ cubic inches) of Silver

$$= 12 \times 12 \times 12 \times \frac{61}{10} \text{ ounces};$$

$$\therefore \text{ Weight of a cubic foot of Iron} = \frac{12 \times 12 \times 12 \times 61}{10} \times \frac{7{\cdot}8}{10{\cdot}5} \text{oz.}$$

$$= \frac{12 \times 12 \times 12 \times 61 \times 7{\cdot}8}{105} \text{ oz.} = 7830 \text{ ounces, nearly}$$

69. [*To find the relation which exists between the Weights, Magnitudes, and Specific Gravities of two substances and of a compound formed of them.*

Let WMS, $W'M'S'$, $W''M''S''$, be the Weight, Magnitude, and Specific Gravity of each of the two substances and of the compound respectively.

Then, it being supposed that the portions of the substances that are combined together lose neither bulk nor weight by being mixed,

$$\text{i. } M'' = M + M'.$$

$$\text{ii. } W'' = W + W'.$$

$$\therefore \text{ Art. 68, iii. } M''S'', \text{ or } (M + M')S'', = MS + M'S'.$$

And it might be shown that similar relations exist between the weights, magnitudes, and specific gravities of the several substances and the compound formed of them, whatever be the number of the simple substances.]

70. [Cor. Let σ, σ', σ'', be the numbers which are attached to the names of the two simple substances and the compound in the "Tables of Specific Gravities," the S. G. (specific gravity) of water being 1.

By Art. 69, $(M + M')S'' = MS + M'S'.$

$$\therefore (M + M')\frac{S''}{S} = M + M'\frac{S'}{S}.$$

But $S' : S$ :: weight of a cubic inch of the second substance : weight of a cubic inch of the first substance.
:: $\sigma' : \sigma$; by foot-note to Art. 67, i.

So $S'' : S :: \sigma'' : \sigma.$

$$\therefore (M + M')\frac{\sigma''}{\sigma} = M + M'\frac{\sigma'}{\sigma}.$$

$$\therefore (M + M')\sigma'' = M\sigma + M'\sigma'.]$$

Cor. In like manner, if W, W', W'' be the weights, in pounds, of the two substances and their compound respectively, it may be proved that

$$\frac{W''}{\sigma''} = \frac{W}{\sigma} + \frac{W'}{\sigma'}.$$

71. PROP. VIII. *When a body of uniform density floats on a fluid, the part immersed : the whole body :: the specific gravity of the body : the specific gravity of the fluid.*

Let a solid of uniform density float on a fluid with M cubic inches of it above, and N cubic inches below, the horizontal plane of the surface.

Let S = S. G. (Specific Gravity) of the solid; S' = S. G. of the fluid.

Then, $(M + N) \times S$ = weight of the solid. Prop. VII.

$N \times S'$ = ... fluid displaced.

But because the body floats, these two weights are equal, Prop. VI.;

$$\therefore N S' = (M + N) S; \text{ and } \therefore \frac{N}{M+N} = \frac{S}{S'},$$

or $N : M+N :: S : S'$;

i. e., The part of it immersed : the whole of the body :: S. G. of the body : S. G. of the fluid. Q. E. D.

72. PROP. IX. *When a body is immersed in a fluid, the weight lost : whole weight of the body :: the specific gravity of the fluid : the specific gravity of the body.*

Let M be the number of cubic inches in a body of uniform density which is wholly immersed in a fluid; S the S. G. of the body, S' the S. G. of the fluid.

Then the pressure downwards of the solid is its weight; and if the solid be removed, and the space it filled be occupied by an equal bulk of the fluid, equilibrium will still exist. And if the fluid so added become solid, the equilibrium will continue, and the pressure upwards of the surrounding fluid will remain the same as before.

Now the pressure downwards of the portion of the fluid that becomes solid is its weight. And as the pressure upwards of the surrounding fluid supports this weight, that pressure must be exactly equal and opposite to it.

The pressure downwards, therefore, before it was immersed, of the original solid, (i. e. its weight MS, Prop. VII.), must, by the solid having been immersed, have been diminished by a pressure upwards, arising from the surrounding fluid, exactly equal to the weight of the fluid displaced,—which weight, by Prop. VII., is equal to MS'.

$\therefore$ Weight lost by the body : the whole weight of the body

:: $MS' : MS :: S' : S$

:: S. G. of the fluid : S. G. of the body. Q. E. D.

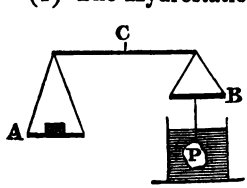
73. [It appears from the proof of the last Proposition, that the pressure of a fluid on a body wholly immersed in it acts vertically upwards, and is equal to the weight of the fluid which the body displaces.

If this pressure be less than the weight of the body—that is, if the S. G. of the fluid be less than that of the solid—the pressure downwards arising from the weight of the solid will be greater than the pressure of the surrounding fluid upwards, and the body will therefore sink to the bottom of the vessel.

But if the pressure of the fluid upwards be greater than the weight of the body immersed,—that is, if the S. G. of the fluid be greater than that of the solid,—the pressure upwards will be greater than the pressure downwards, and the body will therefore rise until the conditions of Proposition VIII. are fulfilled, and will then float.]

74. PROP. X. (1) *To describe the Hydrostatic Balance; and* (2) *to show how the Specific Gravity of a body may be found by it,*—first, *when its Specific Gravity is* greater *than that of the fluid in which it is weighed,—and,* second, *when it is* less.

(1) The Hydrostatic Balance is the common balance with a hook attached to the under part of one of its scales, so that bodies may be weighed either by putting them into the scale, or by suspending them from it and letting them be immersed in a fluid as here represented.



(2) First, let the S. G. (Specific Gravity) of the body be *greater* than that of the fluid.

Since the S. G. of the solid is greater than that of the fluid, the body will sink in the fluid. Art. 73.

Let S = S. G. of the solid, S' = S. G. of the fluid.

W = weight required to balance the body when placed *in* the scale.

W' = weight required when the body is immersed.

$\therefore W - W'$ = diminution of the body's weight in consequence of immersion.

Therefore, by Prop. IX.,

$$W : W - W' :: S : S';$$

$$\therefore S = \frac{W}{W - W'} S':$$

Whence, W and W' being known weights, S may be determined if S' be given.

Next, let the S. G. of the body be *less* than the S. G. of the fluid.

When the body, in this case, is forced under the surface of the fluid, the pressure downwards on it (which is the weight W of the body) being less than the pressure upwards (which is the weight of a quantity of fluid equal in magnitude to the fluid displaced), the body must on the whole be acted on by a pressure upwards which is equal to the weight of the fluid displaced *minus* the weight of the solid.

To determine this pressure let there be taken a body Q,—called the *Sinker*,—of greater S. G. than that of the fluid, and large enough to sink both itself and the body P whose S. G. is required when P is attached to it. Let Q be first immersed by itself in the fluid, and balanced by weights in A. Next, leaving the weights in A, let P be attached to Q and both of them be immersed. The scale A will now preponderate, and to restore the equilibrium let a weight W'' be placed in the scale B.* This weight therefore is equal to the tendency upwards of P when immersed.

* The better way in practice is to restore the equilibrium by removing a weight W'' from the scale A, but the method described in the text perhaps renders the demonstration easier to be understood.

i. e., W'' = weight of a bulk of the fluid equal to that of P *minus* the weight of P (W);

$\therefore$ weight of a bulk of fluid of the same size as $P = W + W''$.

Now since (by Prop. VII.) $W = M \times S$, therefore when the magnitudes of the bodies are the same, $W \propto S$;

$\therefore$ weight of P : weight of an equal bulk of fluid

:: S. G. of solid : S. G. of fluid

or, $W : W + W'' :: S : S'$;

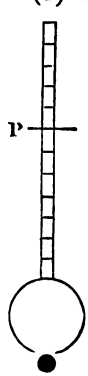
$$\therefore S = \frac{W}{W + W''} S'.$$

Whence, (W and W'' being known weights), S can be found if S' be given.

[It will be observed that it is not requisite that the exact weight should be known of the *Sinker*.]

75. PROP. XI. (1) *To describe the common Hydrometer, and* (2) *to show how to compare the Specific Gravities of two fluids by it.*

(1) The common Hydrometer consists of two spheres attached to each other, and of a long uniform slender stem, which, if produced, would pass through the centers of both of them. The upper sphere is hollow; and the lower, being filled with small shot or mercury, serves as ballast for the instrument, and makes it float steadily in a vertical position when put into a fluid. The stem is graduated by divisions of equal length.



(2) The Hydrometer is made lighter than a bulk equal to that of the instrument of any of the fluids whose Specific Gravities it is employed to compare.

Suppose the bulk of the portion of the stem included between every two graduations to be one four-thousandth part of the bulk of the whole instrument. When the Hydrometer is put into a fluid whose S. G. is S, let 20 divisions be above the surface; and when it is put into a fluid whose S. G. is S', let there be 30 divisions out.

Now the weights of the bulks that are displaced of the two

fluids are the same, each being equal to the weight of the instrument (Prop. VI.). If M and M', therefore, be the magnitudes of the fluids displaced, (by Prop. VII.),

$$M \times S = \text{weight of the Hydrometer} = M' \times S';$$

$$\therefore S : S' :: M' : M$$

$$:: 4000-30 : 4000-20$$

$$:: 3970 : 3980;$$

and the ratio of the Specific Gravities of the two fluids is thus determined.

76. [A mark P is made at the point in the stem to which the instrument sinks in a fluid called "Proof Spirit," which is a mixture consisting of equal *weights*,—(*not* equal *magnitudes*),—of pure Alcohol and of Water. Alcohol being lighter than Water, if a mixture of these two fluids contain a greater weight of the former than it does of the latter, it will be lighter than an equal bulk of Proof Spirit, and the Hydrometer therefore will displace a greater bulk of it than it does of Proof Spirit, that is, it will sink deeper in the mixture than in Proof Spirit. Wherefore the surface of such a mixture will rise to a higher point in the stem than P. In such a case the mixture is said to be "*above proof*."

But if the weight of the Water contained in the mixture be greater than that of the pure Alcohol, the Hydrometer will not sink so low as to the point P, and the fluid is then said to be "*below proof*."]

CHAPTER IV.

ELASTIC FLUIDS.

77. PROP. XII. *Air has Weight.*

This is proved by these experiments.

The weight of a vessel from which the air has been exhausted is found to be *less than* when it was filled with air.

If a bladder be weighed in a vessel from which the air has been exhausted, first when the bladder contains no air, and again after air has been forced into it, a greater weight is required to balance the bladder in the latter case.

Whence it is concluded that "Air has weight."

78. PROP. XIII. *The elastic force of Air at a given temperature varies as the density.*

This is proved by experiment.

Let $ABCD$ be a tube, of which the bore is uniform (its perpendicular section being an area of a square inches), and the legs AB and CD are vertical. The end A of the tube is open, and D is closed.

A quantity of air is retained in the shorter leg by some mercury which rises to E and F in the two legs respectively. There is no air in contact with the mercury in the longer tube. Through E is drawn the horizontal line Ee.

Then, The weight of the mercurial column Fe

= pressure *downwards* at e on a surface a (by Prop. IV., the portion of the tube being vertical),

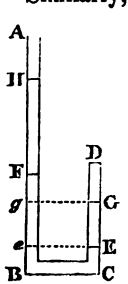
= ,, *upwards* at e ,, ,, ; Prop. I.,

= ,, *upwards* at E ,, ,, ; Prop. II.,

= pressure by the air in DE on the surface a of mercury with which it is in contact;

[because, the whole being at rest, the pressures upwards and downwards on the same horizontal plane must be equal.]

Similarly, when more mercury is poured in, if its surfaces stand at G and H in the two legs, it follows, drawing Gg horizontally through G, that



Weight of the mercurial column Hg = pressure by the air in DG on a portion a of the surface of mercury with which it is in contact.

Now if the lengths ED, Fe, GD, Hg be measured, it is invariably found, however the quantities of air and of mercury used in the experiment be altered, that (the air retaining the same temperature),

$$DE : DG :: Hg : Fe;$$

$\therefore$ content of DE : content of DG :: content of Hg : content of Fe.

But since the same quantity of air is contained in DG that there was in DE,

$\therefore$ Density of the air in DE : density of the air in DG

:: content of DG : content of DE

:: content of Fe : content of Hg

:: weight of mercury in Fe : weight of mercury in Hg,

:: pressure of air in DE on a portion a of surface : pressure of air in DG on an equal portion of surface;—[as shown above.]

Whence it appears that

The pressure which air, at a given temperature, exercises on a given surface varies as the density of the air. Q. E. D.

[NOTE. In the experiment here described the longer leg of the tube has been supposed to be placed in a vessel from which the air has been exhausted, so that there is no pressure at all on the surface of the mercury in the longer leg.

If A communicated with the open air, the surrounding atmosphere would press on the surface of the mercury in AB, and would have to be taken into consideration in estimating the whole amount of pressure sustained by an area a placed at e or g.]

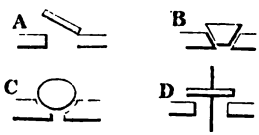
79. PROP. XIV. *The elastic force of the air is increased by an increase of temperature.*

This is proved by experiment.

If a bladder be partially filled with air and then brought near the fire, the included air expands as it becomes heated, and the bladder becomes fully distended. As the included air cools down, the bladder becomes more and more flaccid.

80. DEFS. (1) A VALVE is a kind of door which fits an orifice, such that being pressed by a fluid on one side it opens and allows the fluid to pass through, but which keeps the orifice tightly closed *if the* fluid press on the other side.

[Valves are of various forms,—a flap of leather (*A*) fastened at one edge,—a frustum of a cone (*B*) made of metal,—a sphere (*C*),—or a plate of metal (*D*) with an axis passing perpendicularly through it. By any of these contrivances the flow of a fluid upwards would be prevented only by the weight of the valve; but a rush of fluid from above would carry the valve along with it, and keep the orifice which the valve fits completely closed.

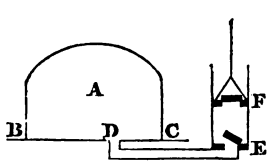


When the fluids employed are very rare,—like air or gas,—the valves are generally made of flaps of oiled or varnished silk, which, being attached at two or three points to the surfaces in which the orifices are situated, are raised by very slight pressures, and so allow fluids of exceedingly small densities to pass under them.]

(2) A PISTON is a plug, mostly cylindrical, which, by means of a rod attached to it, can be made to play within a tube which it fits accurately.

81. PROP. XV. *To describe the construction of the common Air Pump and its operation.*

DESCRIPTION. The Air Pump consists of a glass vessel *A*, called the *Receiver*, made to fit a table *BC* so as to be air tight. A tube *DE* connects the Receiver with a cylinder *EF*, called the *Barrel*. At the bottom of the Barrel there is a valve *E* opening upwards, and a piston *F* (also furnished with a valve opening upwards) plays within the Barrel.



OPERATION. Let the piston *F* be at its highest point, and the instrument filled with air the same as that of the surrounding atmosphere.

When the piston is driven down, the air at first in the barrel is condensed, and its pressure therefore increased. The valve *E*

is kept closed, and the valve in *F* being pressed on the under surface more strongly than on the upper, opens and allows the air in the barrel to escape through it.

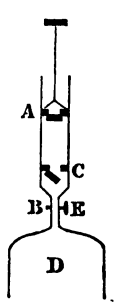
Next, on raising the piston, the external air keeps *F* closed, and, there now being no air in *EF*, the pressure on the under surface of the valve at *E* will open that valve, and allow air from the receiver and pipe to flow into the barrel. The figure represents the instrument during the *ascent* of the piston.

When the piston descends again, the air in *EF* being condensed closes the valve at *E*, and at length opens the valve at *F*, and so escapes; and on the piston reascending more air rushes from the receiver into the barrel.

The process may be repeated until the air in the receiver becomes so rare that its pressure is insufficient to overcome the weight of the valve at *E*.

82. PROP. XVI. *To describe the construction of the Condenser and its operation.*

CONSTRUCTION. The Condenser is a *Barrel AB*, furnished with a piston *A* that has a valve in it opening downwards; at the bottom of the Barrel there is a like valve *C*. The neck of the Barrel communicates with an air-tight vessel *D*, called the *Receiver*.



OPERATION. Let the instrument be filled with common air, and the piston be at its greatest height. On the piston being forced down, the air in the barrel is condensed, and its pressure being therefore increased, (Prop. XIII.), it keeps the valve *A* closed, opens the valve *C*, and is driven into the Receiver. The figure represents the instrument during the *descent* of the piston.

On the piston ascending, the air in the receiver keeps *C* closed, and there now being no pressure on the under surface of the valve

If a bladder be weighed in a vessel from which the air has been exhausted, first when the bladder contains no air, and again after air has been forced into it, a greater weight is required to balance the bladder in the latter case.

Whence it is concluded that "Air has weight."

78. PROP. XIII. *The elastic force of Air at a given temperature varies as the density.*

This is proved by experiment.

Let $ABCD$ be a tube, of which the bore is uniform (its perpendicular section being an area of a square inches), and the legs AB and CD are vertical. The end A of the tube is open, and D is closed.

A quantity of air is retained in the shorter leg by some mercury which rises to E and F in the two legs respectively. There is no air in contact with the mercury in the longer tube. Through E is drawn the horizontal line Ee.

Then, The weight of the mercurial column Fe

= pressure *downwards* at e on a surface a (by Prop. IV., the portion of the tube being vertical),

= ,, *upwards* at e ,, ,, ; Prop. I.,

= ,, *upwards* at E ,, ,, ; Prop. II.,

= pressure by the air in DE on the surface a of mercury with which it is in contact;

[because, the whole being at rest, the pressures upwards and downwards on the same horizontal plane must be equal.]

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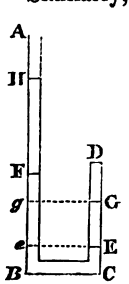

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:: weight of mercury in Fe : weight of mercury in Hg,

:: pressure of air in DE on a portion a of surface : pressure of air in DG on an equal portion of surface;—[as shown above.]

Whence it appears that

The pressure which air, at a given temperature, exercises on a given surface varies as the density of the air. Q. E. D.

[NOTE. In the experiment here described the longer leg of the tube has been supposed to be placed in a vessel from which the air has been exhausted, so that there is no pressure at all on the surface of the mercury in the longer leg.

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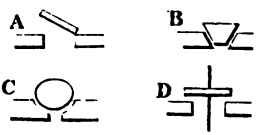
[Valves are of various forms,—a flap of leather (*A*) fastened at one edge,—a frustum of a cone (*B*) made of metal,—a sphere (*C*),—or a plate of metal (*D*) with an axis passing perpendicularly through it. By any of these contrivances the flow of a fluid upwards would be prevented only by the weight of the valve; but a rush of fluid from above would carry the valve along with it, and keep the orifice which the valve fits completely closed.



When the fluids employed are very rare,—like air or gas,—the valves are generally made of flaps of oiled or varnished silk, which, being attached at two or three points to the surfaces in which the orifices are situated, are raised by very slight pressures, and so allow fluids of exceedingly small densities to pass under them.]

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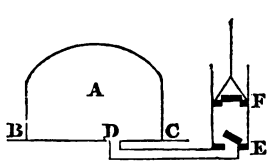
DESCRIPTION. The Air Pump consists of a glass vessel *A*, called the *Receiver*, made to fit a table *BC* so as to be air tight. A tube *DE* connects the Receiver with a cylinder *EF*, called the *Barrel*. At the bottom of the Barrel there is a valve *E* opening upwards, and a piston *F* (also furnished with a valve opening upwards) plays within the Barrel.



OPERATION. Let the piston *F* be at its highest point, and the instrument filled with air the same as that of the surrounding atmosphere.

When the piston is driven down, the air at first in the barrel *is condensed, and* its pressure therefore increased. The valve *E*

is kept closed, and the valve in F being pressed on the under surface more strongly than on the upper, opens and allows the air in the barrel to escape through it.

Next, on raising the piston, the external air keeps F closed, and, there now being no air in EF, the pressure on the under surface of the valve at E will open that valve, and allow air from the receiver and pipe to flow into the barrel. The figure represents the instrument during the *ascent* of the piston.

When the piston descends again, the air in EF being condensed closes the valve at E, and at length opens the valve at F, and so escapes; and on the piston reascending more air rushes from the receiver into the barrel.

The process may be repeated until the air in the receiver becomes so rare that its pressure is insufficient to overcome the weight of the valve at E.

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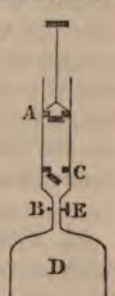
Construction. The Condenser is a *Barrel* AB, furnished with a piston A that has a valve in it opening downwards; at the bottom of the Barrel there is a like valve C. The neck of the Barrel communicates with an air-tight vessel D, called the *Receiver*.



Operation. Let the instrument be filled with common air, and the piston be at its greatest height. On the piston being forced down, the air in the barrel is condensed, and its pressure being therefore increased, (Prop. xiii.), it keeps the valve A closed, opens the valve C, and is driven into the Receiver. The figure represents the instrument during the *descent* of the piston.

On the piston ascending, the air in the receiver keeps C closed, and there now being no pressure on the under surface of the valve

A, the external air opens the valve, and the barrel becomes filled with air, which may be driven into the receiver by forcing down the piston again. And in this manner the condensation of the air in the receiver may be carried on to any extent required.

[The communication between the barrel and the receiver can be cut off at pleasure by means of a stopcock at *E*; and the barrel is made to screw off and on at a point above *E*.]

83. PROP. XVII. *To explain the construction of the common Barometer, and to show that the mercury is sustained in it by the pressure of the air on the surface of the mercury in the basin.*

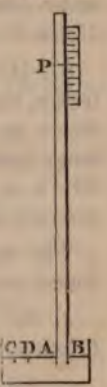
The Barometer is an instrument for measuring the pressure of the air. It consists of a glass tube (see fig. Prop. XVIII.) of uniform bore, closed at one end, and not less than 33 or 34 inches long. This tube is filled with mercury, and the open end, being first stopped with the finger, is placed below the surface of some mercury in a basin, when the tube being held in a vertical position and the finger being withdrawn, the mercury in the tube subsides, and stands at a height above the mercury in the basin which varies on different days from about 28 to 32 inches. A scale of a few inches is attached to the upper part of the tube, to show the height at which the mercury may be standing at any time.

That the column of mercury is supported in the tube by the pressure of the atmosphere appears from the experiment, that, when the whole is put into the receiver of an air pump, the mercury in the tube sinks more and more for every barrel of air that is pumped out, until at length it is all but on a level with the surface of the mercury in the basin.* But on readmitting the air into the receiver the mercury in the tube rises to its former level.

84. PROP. XVIII. *The pressure of the atmosphere is accurately measured by the weight of the column of mercury in the* [vertical tube of the] *Barometer.*

* It has been shown in Prop. XV. that it is not possible to pump *all* the *air out of the receiver*. If it were so, the surface of the mercury in the *tube would then be on* precisely the same level as the surface in the cistern.

Let P be the surface of the mercury in the vertical tube; AB the section of the tube made by the horizontal surface of the mercury in the basin; CD a portion of area in that surface equal to AB.



Then, Weight of mercury contained in PB

= pressure downwards on the area AB, Prop. IV.;

= pressure upwards on AB,

= pressure upwards on CD; since AB and CD are equal areas situated in the same horizontal plane of a fluid at rest, Prop. II.

But the fluid being at rest, the pressure *downwards* on CD, (which arises from the pressure of the air in contact with it), is equal to the pressure *upwards* on it.

Wherefore the pressure of the atmosphere on the area CD is equal to the weight of the column of mercury supported in the [vertical] tube of the barometer, of a base AB which is equal to that area.

Wherefore, "The Pressure, &c." Q. E. D.

85. [Cor. 1. By Art. 62 the pressure of a fluid on a horizontal plane immersed in it was shown to be the weight of a column of the fluid whose base is equal to the area of the plane and whose height is the depth of the plane below the surface of the fluid. Wherefore, the pressure exerted by the atmosphere on such an area,—being measured by the weight of the vertical column of mercury (of equal section) that is supported in the barometer,—is equal to the weight of a column of mercury whose base is equal to the plane acted upon, and whose height is the same as that of the column of mercury supported in the vertical tube of the barometer.]

86. [Cor. 2. The pressure of the air being measured by the weight of the column of fluid which it supports, the barometer might be filled with any fluid whatever. But mercury being by far the heaviest fluid known, the column of mercury required to produce a given pressure is very much shorter than if any other fluid were employed.

For example, if it took 30 inches of mercury to balance the

pressure of the air, then since mercury is 13·6 times as heavy as water (see Note, Art. 67), it would take a column of water $13{\cdot}6 \times 30$ inches, or 34 feet high, to produce the same effect.]

87. [Cor. 3. The mercury in the barometer standing at 30 inches, the pressure of the air on a horizontal square inch of surface is equal to the weight of a column of mercury 30 inches long, and whose base is a square inch; i. e. to the weight of 30×1, or 30, cubic inches of mercury.—*To find how much this pressure amounts to.*

The weight of a cubic foot of Water is found to be 1000 ounces avoirdupois;

$\therefore$ the weight of a cubic foot of Mercury $= 1000 \times 13{\cdot}6$ oz. (Note to Art. 67.)

$\therefore$ (1, or) $\underset{\text{cubic foot.}}{12} \times \underset{\text{cubic inches.}}{12} \times \underset{\text{cubic inches.}}{12} : 30 :: 1000 \times 13{\cdot}6 :$ weight (in ounces) of 30 cubic inches of mercury, which $\therefore = \dfrac{13600 \times 30}{12 \times 12 \times 12}$ ounces $=$ nearly 236 oz., or $14\frac{3}{4}$ lbs.]

And the pressure of the air on a square inch will be greater or less than $14\frac{3}{4}$ lbs. according as the mercury in the barometer stands at a greater or less height than 30 inches.

It appears, then, that every square inch with which the air is in contact is subjected to a pressure of about 14 lbs.; so that a page of a book 6 inches deep by 5 inches wide sustains a pressure not less than $6 \times 5 \times 14$ lbs., or 420 lbs. The reason why the leaf is not torn by this enormous pressure is that the pressure of the air on one side of it is counterbalanced by that on the other side.]

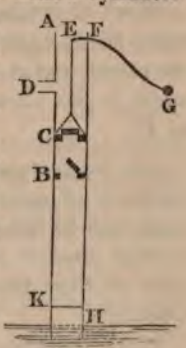
88. [Cor. 4. In Chapters I. and II. the pressure of the air on the surface of the fluid contained in an open vessel has been left out of consideration; in other words, the experiments there described were supposed to be made in the exhausted receiver of an air-pump.

From Art. 86 it appears that when water is the fluid employed, the pressure of the air on its surface is equivalent, (when the mercurial barometer stands at 30 inches), to that which would be *produced* by a head of water 34 feet deep. In estimating, therefore, the pressure on any surface placed at a given depth below

the surface of water, this large additional pressure,—amounting to more than 14 lbs. on a square inch of surface, (Art. 87),—must be taken into account.]

89. Prop. XIX. *To describe the Construction of the Common Pump and its Operation.*

Construction. In the *Common Pump* two hollow cylinders AB and BH, which have the same vertical axis, are connected together, and at their junction a valve B opening upwards is placed. The upper (AB) of these cylinders is called the "*Body of the Pump;*" and in it a piston C, containing a valve opening upwards, plays by means of a rod attached to the end E of a lever EFG whose fulcrum is F. A spout D is placed just above the highest point to which this piston ascends. The lower cylinder (CH), which is called the "*Suction Pipe,*" reaches below the surface H of a well of water.



Operation. Let the piston be at B and the suction pipe full of air. As C is raised, the pressure of the external air keeps the valve in C closed, and a vacuum between B and C being consequently made, the air in HB, pressing against the under surface of the valve at B, opens it, and a portion of the air escapes into BC.

The air therefore which before the ascent of the piston occupied the space BH now occupies the greater space CBH, and so becoming less dense than before exercises a less pressure on the surface of the water at H, Prop. XIII. Wherefore, since the external air continues to exercise the same pressure as before on the surface of the water in the well, it will force up water into the suction pipe to a height K such that the pressure of the air in CK, together with the pressure arising from the column of water KH, produces the same effect on the section of the water in the suction pipe at H, as the external air does on an equal area situated in the surface of the water in the well.

When C has come to the highest point of its ascent, and equilibrium exists between the pressure of the external air on

the surface of the water in the well on the one hand, and the pressure of the air in CK together with that arising from the fluid column KH on the other, the valve B, being equally pressed on its upper and its under surfaces, will fall down by its own weight. The piston C is then let down; the air in CB is condensed until its pressure becomes greater than that of the external air, when it opens the valve in C and escapes.

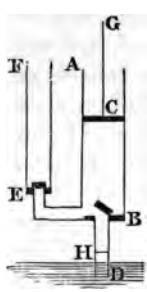
When C is raised again, the same circumstances recur. The water rises a little higher in the suction-pipe at every stroke of the piston, and at last flows through B; and on C descending again, it gets above the valve C, and is brought up to the spout at D.

[As the *average* pressure of the atmosphere will not support a column of water more than 34 feet high, if the valve B be more than 34 feet above the surface of the water in the well, the average pressure of the external air in that case not being sufficient to force the water up so high as B, the pump will not work.

N.B. The figure represents the pump during an *ascent* of the piston; when (air, or water, flowing through it) the valve at B is *open*, and that at C is *shut*.]

90. Prop. XX. *To describe the Construction of the Forcing Pump, and its Operation.*

Construction. The Forcing Pump consists of a *Barrel* AB in which a solid piston C works by means of a rod GC; BD is a *Suction Pipe* reaching below the surface D of a well of water; BE a pipe connecting BC with a vessel EF; at B and E valves are placed, opening upwards.



Operation. Suppose the piston C to be at its greatest height and the pump full of air. When C descends the air in AB is condensed, and its pressure being increased (Prop. xiii.), the valve E is opened, and the greater part of the air that was at first in AB is forced into EF.

On the piston reascending, the air in EBC being now rendered *less dense* than that in EF and BD, the valve E is kept

closed by the external air, the valve B is opened, and a portion of the air which was at first in DB rushes into the Barrel. The external air then forces some water a little way up the pipe (to H suppose), until the pressure on the horizontal section of the suction pipe at D (a pressure which arises from the pressure of the rarified air in HBC together with that of the column of water HD), is equal to the pressure of the external air on an equal area in the surface of the water in the well. When this has taken place there is equilibrium; and the valve B, being pressed equally on both its surfaces, closes by its own weight.

The piston again descends, and the air in AB is driven through E into EF; it ascends again, and more water is forced up the suction pipe: and these operations are repeated until the water rises above B, when on the next descent of the piston *Water* is driven into EF.

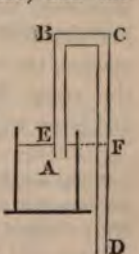
[As in the case of the common pump, unless BD be less than 34 feet, the pressure of the atmosphere upon the surface of the water in the well will not be enough to raise the water above the piston at B; in which case the machine will not work.

N.B. The Figure represents the Forcing Pump during the *ascent* of the piston, when the valve E is *shut*, and the air (or water) is rushing from the suction pipe into the Barrel through the *open* valve B.]

91. Prop. XXI. *To explain the action of the Siphon.*

The Siphon is an instrument for drawing fluids out of vessels. It is a bent pipe or tube, with legs of unequal lengths.

Suppose the Siphon $ABCD$ to be of uniform bore, and the two legs AB and CD parallel and connected together by a pipe BC at right angles to them. Let the instrument, after it is filled with the same fluid as that in the vessel and its ends are stopped, be placed with the legs vertical, and the end A of the *shorter* leg plunged below the surface E of the fluid in the vessel. Let the area of the perpendicular section of the pipe be a square inches, and let F be *in the* same horizontal line as E. Sup-



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This is proved by experiment.

Let $ABCD$ be a tube, of which the bore is uniform (its perpendicular section being an area of a square inches), and the legs AB and CD are vertical. The end A of the tube is open, and D is closed.

A quantity of air is retained in the shorter leg by some mercury which rises to E and F in the two legs respectively. There is no air in contact with the mercury in the longer tube. Through E is drawn the horizontal line Ee.

Then, The weight of the mercurial column Fe

= pressure *downwards* at e on a surface a (by Prop. IV., the portion of the tube being vertical),

= ,, *upwards* at e ,, ,, ; Prop. I.,

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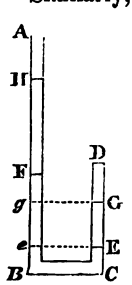
= pressure by the air in DE on the surface a of mercury with which it is in contact;

[because, the whole being at rest, the pressures upwards and downwards on the same horizontal plane must be equal.]

Similarly, when more mercury is poured in, if its surfaces stand at G and H in the two legs, it follows, drawing Gg horizontally through G, that



Weight of the mercurial column Hg = pressure by the air in DG on a portion a of the surface of mercury with which it is in contact.

Now if the lengths ED, Fe, GD, Hg be measured, it is invariably found, however the quantities of air and of mercury used in the experiment be altered, that (the air retaining the same temperature),

$$DE : DG :: Hg : Fe;$$

$\therefore$ content of DE : content of DG :: content of Hg : content of Fe.

But since the same quantity of air is contained in DG that there was in DE,

$\therefore$ Density of the air in DE : density of the air in DG

:: content of DG : content of DE

:: content of Fe : content of Hg

:: weight of mercury in Fe : weight of mercury in Hg,

:: pressure of air in DE on a portion a of surface : pressure of air in DG on an equal portion of surface;—[as shown above.]

Whence it appears that

The pressure which air, at a given temperature, exercises on a given surface varies as the density of the air. Q. E. D.

[NOTE. In the experiment here described the longer leg of the tube has been supposed to be placed in a vessel from which the air has been exhausted, so that there is no pressure at all on the surface of the mercury in the longer leg.

If A communicated with the open air, the surrounding atmosphere would press on the surface of the mercury in AB, and would have to be taken into consideration in estimating the whole amount of pressure sustained by an area a placed at e or g.]

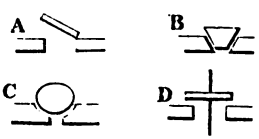
79. PROP. XIV. *The elastic force of the air is increased by an increase of temperature.*

This is proved by experiment.

If a bladder be partially filled with air and then brought near the fire, the included air expands as it becomes heated, and the bladder becomes fully distended. As the included air cools down, the bladder becomes more and more flaccid.

80. DEFS. (1) A VALVE is a kind of door which fits an orifice, such that being pressed by a fluid on one side it opens and allows the fluid to pass through, but which keeps the orifice tightly closed if the fluid press on the other side.

[Valves are of various forms,—a flap of leather (*A*) fastened at one edge,—a frustum of a cone (*B*) made of metal,—a sphere (*C*),—or a plate of metal (*D*) with an axis passing perpendicularly through it. By any of these contrivances the flow of a fluid upwards would be prevented only by the weight of the valve; but a rush of fluid from above would carry the valve along with it, and keep the orifice which the valve fits completely closed.

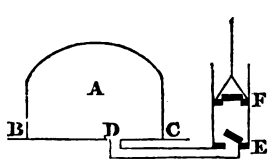


When the fluids employed are very rare,—like air or gas,—the valves are generally made of flaps of oiled or varnished silk, which, being attached at two or three points to the surfaces in which the orifices are situated, are raised by very slight pressures, and so allow fluids of exceedingly small densities to pass under them.]

(2) A PISTON is a plug, mostly cylindrical, which, by means of a rod attached to it, can be made to play within a tube which it fits accurately.

81. PROP. XV. *To describe the construction of the common Air Pump and its operation.*

DESCRIPTION. The Air Pump consists of a glass vessel *A*, called the *Receiver*, made to fit a table *BC* so as to be air tight. A tube *DE* connects the Receiver with a cylinder *EF*, called the *Barrel*. At the bottom of the Barrel there is a valve *E* opening upwards, and a piston *F* (also furnished with a valve opening upwards) plays within the Barrel.



OPERATION. Let the piston *F* be at its highest point, and the instrument filled with air the same as that of the surrounding atmosphere.

When the piston is driven down, the air at first in the barrel *is condensed,* and its pressure therefore increased. The valve *E*

is kept closed, and the valve in F being pressed on the under surface more strongly than on the upper, opens and allows the air in the barrel to escape through it.

Next, on raising the piston, the external air keeps F closed, and, there now being no air in EF, the pressure on the under surface of the valve at E will open that valve, and allow air from the receiver and pipe to flow into the barrel. The figure represents the instrument during the *ascent* of the piston.

When the piston descends again, the air in EF being condensed closes the valve at E, and at length opens the valve at F, and so escapes; and on the piston reascending more air rushes from the receiver into the barrel.

The process may be repeated until the air in the receiver becomes so rare that its pressure is insufficient to overcome the weight of the valve at E.

82. PROP. XVI. *To describe the construction of the Condenser and its operation.*

CONSTRUCTION. The Condenser is a *Barrel* AB, furnished with a piston A that has a valve in it opening downwards; at the bottom of the Barrel there is a like valve C. The neck of the Barrel communicates with an air-tight vessel D, called the *Receiver*.

OPERATION. Let the instrument be filled with common air, and the piston be at its greatest height. On the piston being forced down, the air in the barrel is condensed, and its pressure being therefore increased, (Prop. XIII.), it keeps the valve A closed, opens the valve C, and is driven into the Receiver. The figure represents the instrument during the *descent* of the piston.

On the piston ascending, the air in the receiver keeps C closed, and there now being no pressure on the under surface of the valve

Distance, it should not be left in doubt whether the distance be miles, furlongs, feet, or inches in number. And so if N represent Value, or Mass, or any other quality, precision of reasoning will be impossible unless it be clearly understood from the outset what the things are of which N represents the Number.

The student, therefore, is warned against beginning to solve a problem thus:—

"Let M be the Magnitude of the body:"

the better way would be to say,—

"Let M be the number of cubic inches in the body."

3. After a clear idea has been formed of the manner in which Weight, Magnitude, and other qualities of Matter are expressed, it may, perhaps, be permitted to employ a brief method of describing them, such as, "Let M be the Magnitude of the body"; but for the purpose of *acquiring* definite notions on the subjects treated on in these pages, it is advisable, in solving a problem, that nothing be stated which the student is not able to explain readily.

4. On examining the problems that have been taken from the University Examinations it will be perceived that not many of them are capable of being answered by the direct application of the formulæ established in the Propositions of the Schedule of 1837. They are all, notwithstanding, easily solved by the application of the principles involved in the Demonstrations given in this work of those Propositions.

A problem that can be solved by merely substituting figures in the place of the letters which enter into an algebraic formula is not so much a mental exercise as one in the four elementary rules of arithmetic. And if the solution of a question depend upon a mere substitution in a formula, the question itself may be pronounced to be of little value as a test of the student thoroughly understanding the principles involved in the demonstration of the formula. In framing, therefore, the original questions in the following collection, it has been attempted to make them more easily solved by first principles than by the application of formulæ.

5. The candidate for the degree of B.A. is recommended, when he is called upon to produce his knowledge in the Senate-*house examination*, to draw his figures neatly; both that he may

himself be better able to follow the steps of his demonstrations, and also that the examiner may not have to remark that the Figure and the Demonstration do not agree.

Particular care ought to be taken in drawing the Hydrostatic and other Machines. If a valve be omitted altogether, or if it be represented as shut when it ought to be open, the examiner cannot but decide that the person who could make such an error could not have thoroughly understood the principle of the construction and the operation of the machine.

The slightest deviation ought not to be made in drawing the figure of any machine or instrument, unless the effect of the alteration be perfectly known.

There are no algebraical difficulties involved in the solutions of these examples. It will not even be found necessary in any case to complete the square of a quadratic equation.

i. DENSITIES. Art. 7.

1. *S. H.* 1849. Two bodies, each of uniform density, are of the same size. Show that by weighing them it may be determined whether they are of the same density.

2. Forty cubic inches of a substance weigh 25 lbs., and two cubic inches of another substance weigh 1 lb. Compare the density of the former substance with that of the latter.

3. A cubic foot of a substance weighs 5 cwt., and a cubic foot of another substance weighs 14 qrs. Compare the density of the latter substance with that of the former.

4. Half a cubic foot of water weighs 500 oz. avoirdupois, and a cubic foot of zinc weighs 7000 oz. Compare the densities of water and zinc.

5. A quart of water weighs 2 lbs., and fifty gallons of proof spirit weigh 372 lbs. Compare the density of proof spirit with that of water.

6. A rectangular log of wood is 16 feet long, $2\frac{1}{2}$ feet wide, $1\frac{1}{4}$ feet deep, and weighs 3000 lbs. Compare the density of

the wood with that of a metal a cubic foot of which weighs 480 lbs.

7. Equal bulks of two substances weigh 4 lbs. and 9 lbs. Find the ratio between two bulks of the substances which contain the same quantity of matter.

8. A cubic inch of a substance weighs 280 grains, and a mass of another substance, of twice the density, weighs 400 grains; find its magnitude.

9. A body weighs four times as much as another body which is thrice its bulk. Compare their densities.

10. Half a cubic foot of a substance weighs 2 cwt.; what bulk of another substance that is six times as dense will weigh 3 cwt.?

11. A mass of a substance weighs 8 lbs.; what will be the weight of a mass, twice its size, of a substance whose density is thrice as great?

12. A block of wood weighs 60 lbs.; and a piece of iron, one-third its size, weighs 10 st. 10 lbs. Compare the quantities of matter in the two substances, and their densities.

13. The lengths of three cylinders whose bases are equal, are 1 inch, $4\frac{1}{2}$ inches, and 6 inches, and their weights $\frac{1}{8}$ oz., 4 oz., and $3\frac{1}{2}$ oz. respectively. The density also of the substance of which the shortest cylinder is formed is one-half that of water. Find four whole numbers that bear the same proportion to one another as the densities do of water and the three substances.

ii. LEVERS. Arts. 17—27.

Note.—In the following Examples, unless the contrary be expressly stated, the lever is to be supposed to be straight, and to rest horizontally in equilibrium, and the weights, or other forces, to be applied to it perpendicularly.

1. Half a cwt. acts vertically at the extremity of one of the arms, 10 inches long, of a horizontal straight lever. What number of pounds attached to the extremity of the other arm, 14 inches *long, will balance it?*

2. Two weights, of 5 lbs. and 7 lbs. respectively, keep a horizontal straight lever at rest. The length of the arm at which the larger weight acts is 2 feet 1 inch. Find the length of the other arm.

3. A weight of 14 lbs., suspended 2 inches from the fulcrum of a horizontal straight lever, is balanced by a weight of 8 oz. Find the length of the arm at which the latter weight acts.

4. A straight lever is kept at rest by two forces acting perpendicularly on it on opposite sides of the fulcrum, and at distances from it of 9 inches and 15 inches; the greater force being 6st. 6 lbs., find the other.

5. Divide a cwt. into two portions that will balance on a horizontal straight lever whose arms are 8 feet and 6 feet.

6. *S. H.* 1849. Two weights of 6 lbs. and 9 lbs. respectively, balance at the extremities of a horizontal rod 10 feet long. Find where the fulcrum is placed.

7. On a straight horizontal lever, 20 inches long, two weights, of 24 oz. and of 6 lbs. respectively, balance. Find the position of the fulcrum.

8. *S. H.* 1849. The arms of a horizontal straight lever of the first kind are 12 inches and 18 inches respectively, and the weight which acts at the extremity of the shorter arm is 3 lbs. Find the pressure on the fulcrum.

9. On a lever of the first kind 12 lbs. is supported at a distance of 8 inches from the fulcrum. The pressure on the fulcrum being 14 lbs., find the power and the point of its application.

10. *S. H.* 1849. Two forces, of 3 lbs. and 5 lbs. respectively, balance when they act perpendicularly on a straight lever in opposite directions at the distance of a foot from each other. Find the position of the fulcrum.

11. *S. H.* 1849. At one end of a horizontal straight lever of a given length a force of 3 lbs. acts vertically upwards. At what point must a weight of 12 lbs. be placed to produce equilibrium round a fulcrum put at the other end?

12. On a lever of the second kind, the weight of 24 lbs. is suspended at a distance of 8 inches from the fulcrum: what must

be the length of the arm at which the power, of 64 oz., acts to balance it?

13. A force of 48 oz. acts on a lever of the second class at a distance of a foot from the fulcrum, and is balanced by another of 24 lbs., both forces acting perpendicularly to the lever. Find the distance from the fulcrum at which the second force acts; and also the pressure on the fulcrum, and its direction.

14. Two men of equal height carry 4 cwt. on a pole, the ends of which rest on their shoulders. The weight is placed at a distance of 2-7ths of the length of the pole from one of the men; how many pounds does the other man support?

15. *S. H.* 1849. A lever is held in a horizontal position by two supports that are 5 feet apart, and a weight of 10 lbs. is hung at the distance of 3½ feet from one of the supports. Find the pressure sustained by the other.

16. On a horizontal rod, 45 inches long, whose extremities are supported, where must a weight be placed so that the pressure on the supports may be as 5 to 4?

17. A lever, 7 feet long, is supported in a horizontal position by props placed at its extremities. Where must a weight of 2 qrs. be placed on it so that the pressure on one of the props may be 8 lbs.?

18. The pressure on the fulcrum of a horizontal straight lever of the first class is 36 lbs., and the difference of the lengths of the arms is 6 inches; one-ninth, also, of the pressure on the fulcrum is equal to half of one of the weights which balance each other. Find the weights, and the lengths of the arms at which they act.

19. Two forces, of 8 lbs. and 10 lbs. respectively, acting vertically balance on a horizontal straight lever of the third order; the difference of the lengths of the arms being 2 inches. Find the length of the arm at which the weight acts.

20. A uniform cylinder, 4 feet long, and of which an inch and a half weighs 3 oz., is kept at rest in a horizontal position by a weight of 4 lbs. attached to one end of it. Find the position of the fulcrum.

21. In the last question, if the weight be removed from the

extremity of the arm to a point 2 inches nearer to the fulcrum, what alteration must be made in the position of the fulcrum to restore the equilibrium?

22. Show that two forces acting on a lever, and keeping it at rest, may be exerted in such directions as to render the ratio between them indeterminable.

23. A weight, when it is attached first to one extremity of a straight lever of the first class and then to the other, is balanced by half a pound and 18 oz. respectively. Compare the lengths of the arms of the lever.

24. In a lever where the power and the weight act perpendicularly on the same side of the fulcrum, the weight is given and its point of application, and there is no pressure on the fulcrum. Find the power, and the point of its application.

25. *S. H.* 1849. Explain to what class of levers the handles and body of a wheelbarrow belong.

26. If a straight lever, from the extremities of which weights are suspended, balance in any one position round a fulcrum, it will balance in every other position round that fulcrum.

27. A rod, 14 inches long and 10 lbs. in weight, is joined so as to be in the same straight line with another rod 16 inches long and weighing 8 lbs. Find the point on which they will balance.

28. Two forces acting perpendicularly to a straight lever on the same side of the fulcrum keep it at rest. The difference of the lengths of the arms is 7 inches, and their sum 63 inches. The greater force being $2\frac{1}{2}$ cwt., find the number of pounds in the other.

29. Two forces, of 3 lbs. and 5 lbs., acting perpendicularly to a lever on the same side of the fulcrum, balance. Half the sum of their distances from the fulcrum being 8 inches, find the lengths of the arms.

30. The difference between two weights which keep a horizontal lever at rest is 5 lbs., and the pressure on the fulcrum is 31 lbs. The difference of the lengths of the arms is $4\frac{1}{2}$ inches. Find the weights, and the distance between the points at which they are applied.

31. At one end of a lever, 20 inches long, a weight of 4 lbs. is placed, and is balanced by a weight at the other end. The sum of the weights, together with the pressure on the fulcrum, being 30 lbs., find the second weight, and the distance of the fulcrum from it.

32. Two forces, acting perpendicularly and on the same side of the fulcrum, keep a lever at rest. The distance between their points of application is 4 feet 8 inches, the difference of the forces 16 lbs., and the sum of the forces, together with the pressure on the fulcrum, is 60 lbs. Find the forces, and the distance from the fulcrum at which the lesser force acts.

33. A horizontal uniform cylinder, of which a portion 6 inches long weighs 8 lbs., is $1\frac{1}{2}$ yard in length. A weight of 18 lbs. being attached to one of its extremities, find the position of the fulcrum upon which the whole balances.

34. A rod of platina and another of silver are of the same diameter, and being joined together with their axes in the same straight line, balance in a horizontal position on their junction. Given that the length of the platina rod is 9 inches, and that the density of silver is $\frac{48}{100}$ times that of platina, find the length of the silver rod.

35. Two cylinders, each of uniform density and of the same diameter, when joined with their axes in the same straight line, balance horizontally on a fulcrum placed at their junction. Their lengths are 9 and 16 inches respectively, and 5 inches of the shorter cylinder weigh 8 lbs. How many ounces will 10 inches of the other cylinder weigh?

36. There is a cylinder 2 feet long, and whose base is a square inch. From one of its ends a hole is bored $\frac{3}{4}$ of a square inch in area, its axis being the same as that of the cylinder. What must be its length so that the solid may balance horizontally in a fulcrum placed under the point where the bore ends?

37. There is a rectangular parallelopiped, 18 inches long, whose ends are squares of which a side is an inch. By taking shavings, each a hundredth part of an inch thick, from two-thirds of the length of one of the faces, the body is made to balance on the line from which the shavings begin. Find the *number of the* shavings.

38. Two uniform prisms, whose bases are similar and equal, are joined with their axes coincident, and a fulcrum is placed at their junction. The length of one of the prisms is 8 inches, and 2 inches of it weigh 3 lbs. The length of the other is 13 inches, and 3 inches of it weigh 2 lbs. Show (1) which of the arms will preponderate, and (2) what length must be cut off that arm and joined to the extremity of the other so that equilibrium may take place round the fulcrum.

39. The Weight (W) being given in each of the three kinds of levers, show within what limits the magnitude of the pressure on the fulcrum will lie.

40. *S. H:* 1850. Two given forces, P and Q, acting at the extremity of a rod moveable freely round its other end, keep it at rest. The direction of one of the forces being given, show how that of the other may be found.

41. *S. H:* 1849. A uniform heavy rod is supported by two equal strings attached to its extremities and to a fixed point, the rod and the strings forming an equilateral triangle. Compare the tension of one of the strings with the weight of the rod.

42. *S. H:* 1849. The directions of two forces, P and Q, which act on a bent lever and keep it at rest, make equal angles with the arms of the lever, which are 6 and 8 inches respectively. Find the ratio of Q to P.

[By a "bent lever" is meant one whose arms are straight and are inclined at an angle to each other.]

iii. COMPOSITION AND RESOLUTION OF FORCES.

ARTS. 28—32.

1. Three forces, of 4 lbs., 2 lbs., and 3 oz. respectively, act upon a point in the same direction, and in the opposite direction forces of 8 oz. and 5 lbs. act. What single force will keep the point at rest?

2. *S. H:* 1849. Two forces, of $4\frac{1}{2}$ lbs. and $3\frac{1}{2}$ lbs., applied at a point, have a resultant of 8 lbs.; in what direction do they act?

3. Three equal forces act on a point and keep it at rest. Find the angle at which one is inclined to another.

4. Find the magnitude and the direction of the resultant of two given equal forces which act on a point at right angles to each other.

5. If the sides AB, CA of a triangle ABC represent in magnitude and direction two forces which act on the point A, draw the line which represents the magnitude and direction of their resultant.

6. If two sides, AB and BC, of a triangle ABC, represent in magnitude and direction two forces that act on the point C, draw the line that will represent their resultant in magnitude and in direction.

7. *S. H:* 1849. Two forces, whose magnitudes are as 3 : 4, acting on a point at right angles to each other, produce a resultant of 15 lbs. Find the forces.

8. Two forces, of 5 lbs. and 12 lbs., act at right angles upon a point; find the magnitude of their resultant.

9. Forces represented in magnitude and direction by the sides AB, AC, BC of the triangle ABC, act upon the point C. Draw the line which will represent their resultant in magnitude and in line of action.

10. If at a point A four forces be applied which are represented in magnitude and direction by the sides of the quadrilateral figure $ABCDA$ taken in order, the point will remain at rest.

11. *S. H:* 1849. If three forces acting on a point keep it at rest, when their intensities are doubled they will still keep it at rest if their directions be not changed.

12. Given the lines AB and AC, which represent in magnitude and in line of action the resultant and one of the two forces (acting at A) of which it is compounded, draw the line which represents the other component force in magnitude and in line of action.

13. Forces represented in magnitude and direction by the sides AB, CB, CA of the triangle ABC, act on the point A. Required to draw the line that will represent the magnitude and the direction of the force that will keep them at rest.

14. S. H: 1850. Let ABC be a triangle, and D the middle

point of BC. If the three forces represented in magnitude and direction by AB, AC, DA act upon the point A, find the magnitude and direction of the resultant.

15. A force of 10 lbs. acting perpendicularly on the arm of a lever keeps it at rest. This force is removed, and the equilibrium is maintained by another force, which is applied at the same point in a line which makes half a right angle with the lever: find its magnitude, and show what other effect it has besides preventing the lever from moving round the fulcrum.

iv. VELOCITIES. ART. 43.

NOTE.—In the following Examples, unless it be intimated otherwise, the motion is supposed to be uniform.

1. A railway train goes at the rate of 30 miles an hour. What is its velocity, estimated by the number of feet traversed in a second?

2. A race of 2 miles, 3 furlongs, and 62 yards was run with a uniform speed in 4 minutes and 12 seconds. What was the velocity?

3. A body moves through 40 feet in 3 seconds, and another body moves through 25 yards in 6 minutes. Compare their velocities.

4. A body moves with a velocity of 30. With what velocity must another body move which starts from the same point 3 minutes after the former, and overtakes it in 10 minutes?

5. For 6 seconds a body moves with a velocity of 10 feet, and for the next 9 seconds with a velocity of 15 feet. What uniform velocity would have carried it over the same space in the same time?

6. In 12 minutes a man walks a mile. For the first 3 minutes his velocity is 5, and for the last 5 minutes it is 10; what is his velocity during the rest of the time?

v. MECHANICAL POWERS. Arts. 33—45.

1. The weight that is sustained on an inclined plane by a power that is parallel to the plane is 16 lbs., and the pressure on the plane is 3 lbs. Find the power, to two places of decimals.

2. Two inclined planes of the same height are placed together, height to height, their lengths being 2 feet and 16 inches, and two weights resting upon them balance each other by being connected by means of a string which passes over a pulley, the portions of the string being parallel to the planes. The weight that rests on the longer of the planes being 42 lbs., find the other weight.

3. In a system of three pulleys, each of which hangs by a separate string, the strings being parallel, the power, which is half a cwt., rests upon an inclined plane, the height of which is half its length, by means of a string that passes round a pin, the string, on leaving the pin, being parallel to the plane. Find the weight.

4. A heavy body is supported on an inclined plane whose length is ten times its height, by a string which is parallel to the plane and is attached to the end of a horizontal straight lever, at the other end of which a ton is placed. The length of the arm to which the string is attached being seven times that of the other, find the weight of the body that rests on the plane.

5. A weight of 90 lbs. is supported on an inclined plane by a string, parallel to the plane, which is fastened to the extremity of one of the equal arms of a straight horizontal lever. The weight attached to the other end of the lever being 40 lbs., compare the height of the plane with its length.

6. The weight sustained on a system of two moveable pulleys, each pulley hanging by a separate string and the strings being parallel; the string that passes round the pulley to which the weight is attached is fastened to one end of a horizontal straight lever of the first order, to the other end of which the string is attached that passes round the last moveable pulley. Find the position of the fulcrum of the lever.

7. *S. H: Examination for Honours*, 1850. In the wheel and *axle, if the axis* round which the machine turns coincide with

that of the axle but not with that of the wheel, show what are the greatest and least values of the ratio of the power to the weight, the radius of the axle being 6 feet, and that of the wheel being 15 feet, the axis of the machine also being a yard distant from that of the wheel.

8. *S. H:* 1849. A wheel and axle is applied to sustain a weight upon an inclined plane, the string being parallel to the plane. Find the condition of equilibrium.

9. *S. H:* 1848. A weight of 100 lbs. is suspended from the block of a single moveable pulley, and the end of the string in which the power acts is fastened at the distance of 2 feet from the fulcrum of a straight horizontal lever, 5 feet long, of the second class. Find the force which must be applied perpendicularly at the extremity of the lever to preserve the equilibrium.

10. In a system of three pulleys, where each pulley hangs by a separate string, a weight of W lbs. is suspended at the first pulley, and one of $3W$ lbs. at the second. Find the power.

11. There is a system of three pulleys, each of which hangs by a separate string, and the power, which is a heavy body weighing 25 lbs., rests on an inclined plane, with the string attached to it—by passing round a pin—parallel to the plane. The height of the plane being three quarters of its base, find the weight that is supported.

12. The system of pulleys represented in the figure to Art. 33,—there being two pulleys at the lower block,—is suspended, by a string attached to the higher block, from the arm of a horizontal lever of the second order, and is balanced by a power of 10 lbs. acting vertically. The weight at the lower block of pulleys is 1 cwt., and the length of the shorter arm of the lever is 8 inches; find the length of the other arm;

(1.) When the tension of the string at the point P in the figure is produced by a heavy body that hangs freely;

(2.) When the tension is produced by the string being held by a person who stands on the ground;

(3.) When the motion of W in the system of pulleys is prevented by the string being fastened to a point in the circumference of the lower block.

13. An incline on a railway is 1 in 50, and a train of 70 tons in weight is kept at rest upon it by a rope which is at first parallel to the way, and at the top of the incline wraps round a vertical cylinder that is kept at rest by three horizontal spokes. The diameter of the cylinder is 18 feet, and the length of each of the spokes, measured from the centre of the axis of the cylinder, is 7 yards. Find the power which is applied horizontally at the extremity of each spoke, and at right angles to it.

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vi. CENTER OF GRAVITY. ART. 46—54.

1. *S. H:* 1849. Three weights, of 1 lb., 2 lbs., and 3 lbs. respectively, are placed along a straight line at equal distances of a foot each. Find their C. G. (Center of Gravity.)

2. *S. H:* 1849. A body may balance in two positions round a line that passes through it without the C. G. being in that line; but if it balance in three positions the C. G. must be in the line.

3. The C. G. of a parallelogram is the intersection of its diagonals. The C. G. of a circle is its center.

4. *S. H:* 1849. Construct for the C. G. of two of the sides of a triangle.

5. If an angle A of a triangle ABC be joined with the bisection (F) of the side opposite to it, and G (which is in AF) be the C. G. of the triangle, then $AG = \frac{2}{3} AF$.

6. A side and the C. G. of a triangle being given, construct the triangle.

7. *S. H:* 1849. Find the C. G. of three equally heavy points placed at the three angles of a triangle. Show also that their C. G. coincides with that of the triangle.

8. Weights of 1 lb., 2 lbs., and 4 lbs. being placed at the angles of a given horizontal triangle that is itself without weight, construct for the point on which the triangle will balance.

9. *S. H:* 1849. On a given horizontal base to construct a triangle such that the vertical line through its C. G. shall pass *through* one extremity of the base.

10. S. H: 1850. Find, by construction, the common C. G.

of a triangle ABC and of a heavy body placed at one of its angles, the weight of the heavy body being seven times that of the triangle.

11. If three weights, P, Q, R, be respectively placed at points A, B, C in a horizontal straight line OM, and be situated on the same side of O,—A being the nearest to O, and C the most remote from it,—then the C.G. (G) of the weights is determined by the equation $(P+Q+R).OG = P.OA + Q.OB + R.OC$.

12. If a triangle be bisected by a line parallel to the base, and g and G (the centers of gravity of the triangle cut off and the whole triangle) be joined, and the line g G be produced to a point H such that G $H = g$ G, then H is the C. G. of the figure formed of the whole triangle less the triangle cut off.

13. One-third of a parallelogram is cut off by a line parallel to one of its diagonals. Construct for the C. G. of the remainder.

14. Weights of 2 lbs. each are placed at the extremities of a side—which is a foot in length—of a horizontal square, and weights of 4 lbs. each at the other corners of the square. Find the distance of the C. G. of the whole from either of the lesser weights.

15. From a square $A\,B\,C\,D$ whose diagonals intersect in E, the triangle $A\,E\,B$ is taken away. Construct for the C. G. of the remainder.

16. A square being divided in the same manner as in the last example, one of the triangles ($A\,E\,B$) is folded back round the line $A\,B$, until it is in the same plane as the rest of the square. Find the C. G. of the four triangles.

17. *S. H:* 1850. Show that a parallelopiped, whose vertical side is a rhombus, will always stand upon a horizontal plane.

HYDROSTATICS.

vii. PRESSURE OF NON-ELASTIC FLUIDS.

ARTS. 58—65.

1. If a triangle be placed horizontally in a fluid, it will remain at rest if its Center of Gravity be supported.

2. Find the pressure on the horizontal base of a vessel whose sides are vertical and which is filled with fluid to the depth of 6 inches; a cubic inch of the fluid weighing an ounce and a half, and the area of the base being 64 square inches.

3. *S. H:* 1850. Two rectangular planes whose lengths are 4 inches and 6 inches, and breadths 3 inches and 4 inches, are placed horizontally 4 feet and 2 feet respectively below the surface of a fluid. Compare the pressures on them.

4. A vessel, with vertical sides and horizontal base, which contains fluid, is placed in the scale of a balance, and is poised by weights in the other scale; a person, without touching the vessel, then holds his hand in the fluid. Show whether this has any effect on the equilibrium of the balance.

5. *S. H:* 1850. If a solid be put into a fluid contained in a vessel, whose sides are vertical and bottom horizontal, and floats, explain what additional pressure (if any) there will be on the base.

viii. SPECIFIC GRAVITIES. ARTS. 66—75.

NOTE.—In the following Examples, whenever a mixture is supposed to be made of different substances,—fluid or solid,—unless it be expressly stated otherwise, the Weight and the Volume of the Compound are to be considered respectively equal to the sum of the Weights and the sum of the Volumes of the substances of which it is composed.

Some substances,—Water and Alcohol for instance,—lose bulk *when mixed together,* and so form a mixture that is of a greater

S. G. than if there had been no diminution of bulk. If equal weights of Water (S. G. 1) and of Alcohol (S. G. ·825) were mixed together without bulk being lost, the S. G. of the mixture (Proof Spirit) would be ·904, and not ·930, the S. G. given in the foot-note to Art. 67.

In the following Examples, if the S. G. of a substance be given in figures, the figures are the numbers given in the "Tables of Specific Gravities," explained in the foot-note to Art. 67.

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1. *S. H:* 1849. A cubic inch of iron weighs 4½ oz., and a cubic foot of water 1000 oz., avoirdupois. Find the S. G. of iron.

2. Fourteen cubic feet of granite weigh a ton. Find the *S. G* of granite, a cubic foot of water weighing 1000 oz. Find also the *S. G.* of earth, a cubic yard of which weighs a ton.

3. Twenty pints of a fluid weigh as much as 22 pints of another. Compare the S. G. of the former fluid with that of the latter.

4. The weight of 540 cubic inches of a substance is 10 lbs., that of 360 cubic inches of another is 40 oz. The S. G. of the second substance being 3, what is that of the other?

5. Ten cubic inches of a substance weigh 4 lbs., 4 cubic inches of another weigh 10 lbs. How often does the S. G. of the latter substance contain that of the former?

6. *S. H:* 1849. A cubic foot of water weighs 1000 oz. Find the length of an edge of a cubical block of a substance which weighs 2000 tons, the S. G. of the substance being 1·12 times that of water.

7. The S. G.s of two fluids are ·78 and ·66. What weight of the latter fluid will be of the same bulk as 91 lbs. of the former?

8. The S. G. of tin being 7·2, find how many cubic inches of copper (S. G. 9) will weigh as much as a cube of tin whose edge is 4 inches. Find also the S. G. of the compound metal formed by fusing the lumps of tin and copper together.

9. Ninety pounds of silver are of the same bulk as 75 lbs. of a mixed metal. How many times does the S. G. of silver contain that of the mixed metal?

10. The S. G.s of two substances are as 45 to 81. The weight of 240 cubic inches of the former is 18 lbs.; what will be the weight of 360 cubic inches of the latter?

11. *S. H:* 1849. A pint of water is mixed with a quart of milk. The S. G. of milk being 1·03, find the S. G. of the mixture.

12. Six pints of fluid (S. G. ·925) are mixed with two quarts of water. Find the S. G. of the mixture.

13. In 50 cubic yards of rock, whose average S. G. is 1·42, there enter 32 cubic yards of a substance whose S. G. is 1·24. Find the S. G. of the remainder of the rock.

14. Find the S. G. of a mixture composed of equal bulks of two fluids whose S. G.s are ·9 and 1·2.

15. Five pints of a fluid (S. G. 1·04), 2 quarts of another (S. G. ·97), and 4 gallons of a third (S. G. 1·1), are mixed together; find the S. G. of the mixture.

16. Portions of two given fluids that are not of the same S. G. form a mixture of which the S. G. is half the sum of the S. G.s of the fluids composing it. Compare the volumes of the fluids that are employed.

17. If W, W', W'' be the weights of two substances and of a compound formed of them, whose S. G.s, as given in "Tables of Specific Gravities," are σ, σ', σ'' respectively, show that

$$\frac{W''}{\sigma''} = \frac{W}{\sigma} + \frac{W'}{\sigma'}$$

18. Find the S. G. of a mixture formed by mixing equal weights of water and of a fluid whose S. G. is ·825.

19. Nine pounds of fluid S. G. 1·05 are mixed with 7 lbs. of water. Find the ratio of the S. G. of the mixture to that of water.

20. Eight ounces of salt being mixed with 2 quarts of water (without increasing its volume), make a brine in which an egg just floats. The weight of a pint of water being 1 lb., find the S. G. of the egg.

21. There are mixed together 8, 12, and 20 lbs. of three fluids *respectively, and the* S. G.s of the first fluid, the second fluid,

and of the mixture are 1·6, ·8, and 1·2 respectively. Find the S. G. of the third fluid.

22. Five gallons of a fluid of S. G. ·98 are mixed with 6 gallons of a fluid of S. G. 1·6 without any weight being lost, and the S. G. of the mixture is 1·3. Has volume been either gained or lost?

23. A quart of fluid of S. G. ·8 is mixed with a gallon of fluid of S. G. 1·05, but from a chymical union taking place the volume of the mixture is one hundredth part less than the sum of the volumes of the portions of the fluids composing it, though no weight is lost. Find the S. G. of the mixture to four places of decimals.

24. Twenty ounces of a substance when plunged into a cylindrical vessel whose base is horizontal and which contains a pint of water, increase the depth of the water from 4 inches to $4\frac{1}{2}$ inches. The weight of a pint of water being 1 lb., find the S. G. of the substance.

25. If $W = \mu\sigma$, where W expresses the weight of the body in ounces, and σ the number given in the "Tables of Specific Gravities" as the S. G. of the substance, what does μ represent?

26. A rectangular tunnel, half a mile long, 8 feet wide, and 12 feet high, is to be made through a rock of S. G. 1·4, and the excavated rock is all to be brought to the same end of the tunnel. The expense of quarrying is 20*s.* for every yard in length, and the rock costs for removing three farthings per ton for every yard it is carried. The weight of a cubic foot of water being 1000 oz., find what the tunnel costs.

27. The S. G.s of platina, iron, and water being 21·5, 7·8, and 1 respectively, find the numbers to three places of decimals which will represent the S. G.s of platina and water when the S. G. of iron is taken to be 1.

28. A vessel, whose sides are vertical and base horizontal, contains three quarts of fluid, the depth of which is 10 inches. When a piece of copper of S. G. 8·9 is immersed in the fluid, the surface of the fluid rises to $11\frac{1}{2}$ inches above the base. Find the weight of the piece of copper, the weight of a pint of water being 1 lb.

29. *S. H:* 1850. A solid, whose weight is 6 lbs., floats on a

fluid so that the weight of the portion above the surface is 2 lbs. Compare the S. G.s of the fluid and the solid.

30. *S. H:* 1849. A cube of cork whose edge is a foot, floating vertically in water sinks to the depth of 2·88 inches. Find its S. G., that of water being 1.

31. A solid, of 15 cubic inches, and whose S. G. is 0·6, floats on water. What is the content of the portion of it below the surface?

32. A block of wood of S. G. ·64 floats on a fluid with 5-9ths of it below the surface; find the S. G. of the fluid.

33. *S. H:* 1849. A prismatic solid, 10 inches long, floats vertically with 3-5ths of its length immersed. Compare the S. G. of the solid with that of the fluid, and if the solid float vertically after having been shortened 4 inches, find what length of it will then be above the surface of the fluid.

34. *S. H:* 1850. A solid, whose weight is 18 lbs., floats on a fluid whose density is three times that of the solid. Find the weight of the portion of the solid which remains above the fluid.

35. *S. H:* 1849. A cubical block, whose edge is 3 inches, floats with a side 1 inch above the surface, and if an ounce weight be put on the block it becomes just even with the surface of the fluid. Find the weight of the solid.

36. *S. H:* 1849. The weight of a cubic foot of oak is 75·6 lbs., and when immersed in water it weighs 12·6 lbs. Find its S. G.

37. When to one of the faces of a cube of wood whose edge is 1 foot, a plate of copper of the same area is attached, the whole is found to be of the same S. G. as water. The S. G.s of the wood and of the copper being ·8 and 9, find the thickness of the plate.

38. Given that a cubic foot of water weighs 1000 oz., find how many nails, of 24 to the pound, must be driven up to their heads in a cubic foot of wood of S. G. ·75, so that the whole may be of the same S. G. as water.

39. *S. H:* 1849. A piece of wood which weighs 3 lbs. floats *in water, and* the S. G. of the wood : S. G. of water :: 3 : 4. *What weight must* be placed on the wood so as just to sink it?

40. The weight of a piece of wood is 45 lbs., and when it is put into water 3 lbs. are required to sink it even with the surface. Find the S. G. of the wood.

41. A body floats with 3 lbs. of it above the surface of the fluid, and when another body, half its size, and of S. G. twice as great, is placed upon it, it sinks even with the surface. Find the weight of the former body.

42. *S. H:* 1849. A weight of 4 lbs. when placed on a piece of wood whose S. G. : S. G. of water :: 3 : 5, just causes it to be immersed. Find the weight of the wood.

43. A block of wood weighs 10 lbs. in air, and when put into a vessel of water it sinks, pressing the bottom of the vessel with a force of 2 lbs. Find what proportion of the block must be hollowed out,—the orifice of the aperture being closed with wood of the same kind,—so that it may float just even with the surface of the water when it is held down with a force of 2 lbs. Find also the S. G. of the wood.

44. *S. H:* 1850. A piece of wood floats with 1-3rd of its bulk out of water; and a stone, whose bulk is 1-8th that of the wood, when placed upon it, just sinks the wood even with the surface of the water. Find the S. G. of the stone.

45. *S. H:* 1849. *Examination for Honours.* A cylindrical vessel of horizontal base contains water. If a cubic foot of cork of S. G. ·24 be allowed to float on the water, find the additional pressure on the base, the weight of a cubic foot of water being 1000 oz.

46. A body, whose volume is 250 cubic inches, weighs 6 lbs. in a fluid 50 cubic inches of which weigh $2\frac{1}{2}$ lbs. What will the body weigh out of the fluid?

47. The edges of a rectangular block of marble are 4, 7, and 12 feet, and a piece of the marble that weighs 8 oz. in air, weighs $1\frac{1}{4}$ oz. in water. Find the weight of the block, it being given that a cubic foot of lead (S. G. 11·2) weighs 700 lbs.

48. *S. H:* 1850. Ninety-five ounces of gold and 99 ounces of lead balance when weighed in water. The S. G. of gold being 19, find (1) the S. G. of lead, and (2) the S. G. of an ounce of gold and an ounce of lead melted together.

49. *S. H:* 1849. If the S. G.s of iron and of gold be 8 and 19 times respectively that of water, find the weight in water of a substance combined of 1 lb. of iron and 1 lb. of gold.

50. A piece of iron weighs 12 lbs. in water, and when a piece of wood that weighs 5 lbs. is attached to it, the two together weigh 9 lbs. in water. Find the S. G. of the wood.

51. A piece of iron weighs 10 lbs. in air, and when it is attached to a piece of platina (heavy enough to sink it) which has been immersed in mercury and balanced, the weight that restores the equilibrium is 7 lbs. 6 oz. The S. G. of mercury being 13·6, find that of iron.

52. A body rests, wholly immersed, between two fluids which do not mix, M cubic inches of it being surrounded by the lighter fluid and N by the heavier. Find the ratio $M : M+N$ in terms of the S. Gs. of the two fluids and of that of the solid.

53. *S. H:* 1849. *Examination for Honours.* A body floats in a mixture of two fluids with a volume a immersed. If one-half of the mixture be removed, and its place supplied by an equal magnitude of the lighter fluid, a volume $a+b$ is immersed. Determine the ratio of the volumes of the fluids contained in the original mixture.

54. A cylinder of metal, of S. G. 12·4 and 14 inches long, floats wholly immersed, with its axis vertical, in a vessel filled with mercury (S. G. 13·6) and water. Find the length of the portion of the cylinder that is surrounded by the water.

55. If W, W' be the weights in air, and w, w' the weights in water of two substances, the former of which is of a greater density than the latter, show whether $W \div w$ be less or greater than $W' \div w'$.

56. Three masses, of gold, silver, and a compound of gold and silver, each weigh n ounces in air, and p, q, r ounces respectively when immersed in water. Determine the weight of the gold in the compound.

57. Three masses, of gold, silver, and a compound of gold and silver, weigh respectively P, Q, and R ounces in air, and p, q, *and* r ounces in water. Find the ounces of gold in the *compound.*

58. Taking the quantities P, Q, R, and p, q, r, from the last question, show what is the order of magnitude of the quantities $p \div P$, $q \div Q$, $r \div R$.

59. There are masses of two metals which weigh 51 and 39 oz. respectively, and a mass containing certain proportions of the metals weighs 45 oz., and each of the three bodies loses the same weight when immersed in the same fluid. Find the number of ounces of the first metal which enters into the compound.

60. A vessel which contains fluid is balanced in a pair of scales. A heavy body is then suspended in the fluid as represented in the figure to Art. 72. What effect (if any) is produced in the equilibrium of the scales?

61. A uniform cube floats with a face, of which the diagonal is 10 inches, horizontal, two weights being placed on the face,—one, of 8 lbs., being at an angle of it; find the position of the second weight, which is 20 lbs.

62. A cubic foot of water, weighing 1000 oz., is put into a vessel which is of the form of two cylinders placed, with their axes vertical and coincident, one upon the other, and communicating with each other. The areas of the bases of the lower and the upper cylinders are 54 and 24 square inches respectively, and the height of the lower cylinder is 18 inches. Show that the pressure on the base of the vessel is $1546\frac{2}{3}$ oz.

63. *S. H:* 1849. Two vertical tubes, of equal bore, are connected by a horizontal tube 2 inches long. Supposing 12 inches in length of mercury to be poured into one tube, and 26 of water into the other, find the altitudes of the water and the mercury in the two branches, the S. G. of mercury being supposed to be 13 times that of water.

ix. ELASTIC FLUIDS. Arts. 77—92.

1. *S. H:* 1849. Explain why a balloon rises, and why the higher it gets the slower it rises.

2. *S. H:* 1849. If the mercury in a barometer be at 30 inches, what is the greatest vertical length of the suction pipe of a common pump that will pump up mercury?

3. A pump will raise water 33 feet; how high will it raise oil of the S. G. ·88?

4. *S. H:* 1849. If 13 inches of water be inserted in the tube of a barometer upon the mercury, what will be the altitude of the upper surface of the water when the common barometer stands at 30 inches, the S. G. of mercury being 13? And how much will the top of the water fall when the mercurial barometer sinks 1 inch?

5. *S. H:* 1849. The height of a mercurial barometer being 30 inches, required the height of a barometer of which the column above the cistern contains equal weights of mercury S. G. 13·6, and of a fluid S. G. ·93.

6. *S. H:* 1849. The height of the mercurial barometer being 30 inches, required that of a barometer filled with equal lengths of mercury and proof spirit, the S. G. of mercury being 13·6, and that of proof spirit being ·93.

7. *S. H:* 1850. What would be the effect if a small quantity of air were introduced into the tube of a common barometer?

8. Two substances of different specific gravities balance when the barometer is at 28 inches; which of them, taking the weight of the air displaced into consideration, will preponderate when the barometer is at 30 inches?

Hence also show whether the buyer or the seller of diamonds that are bought by weight loses when the mercury stands in the barometer at a height above the average; the weights employed being supposed to be of a greater S. G. than diamond.

9. *S. H:* 1850. A piston fits a hollow cylinder, whose height is 7 inches and base 1 square foot. Supposing the cylinder originally filled with common air, the pressure of the atmosphere to be 14 lbs. on a square inch, and the piston to sink 1 inch in the cylinder, find the weight of the piston.

10. The tube of a mercurial barometer is vertical, and of uniform bore. On a syringeful of air being introduced into the upper part of the tube the mercury falls 1 inch; and it falls eight-tenths of an inch more when another syringeful is introduced. The mercury in the cistern being kept at the same level *throughout, find* the length of that portion of the tube that was *originally a* vacuum.

11. *S.H:* 1850. What would be the effect if a small quantity of air were introduced into the vacuum of a thermometer? and why would it derange the thermometer more than if the sealed end were open to the atmosphere?

12. If the distance between the two points on the stem of a common hydrometer to which the instrument sinks in two fluids of different S. Gs. be divided into 100 parts, the number of those parts that are immersed when the instrument is placed in a mixture of the two fluids will indicate the parts (in weight) per cent. of the lighter fluid which the mixture contains.

13. A hydrometer sinks to a certain point in a fluid whose S. G. is s, and the part of the stem that is not immersed has to be loaded with m grains to sink the instrument to the same point in a fluid whose S. G. is s'. Find the number of grains with which the instrument must be loaded to sink it to the same point in a mixture of the two fluids which contains n parts per cent., by measure, of the heavier fluid.

14. A condenser at first is full of air the same as that of the surrounding atmosphere, and the content of the barrel is one-tenth that of the receiver. From a flaw in the receiver it will only sustain a pressure equal to three-fourths that of the atmosphere. During what descent of the piston will the receiver burst?

15. In the last question, how much of its last descent can the piston perform without the receiver bursting?

x. GRADUATIONS OF THERMOMETERS.

ARTS. 93, 94.

1. Find the degrees of the Centigrade that show the same temperatures as 77°, 23°, and 32° below zero, of Fahrenheit.

2. Find the degrees of Fahrenheit that show the same temperatures as 50°, 110°, — 10°, — 20°·5 of the Centigrade.

3. *S. H:* 1849. Show that 95° of Fahrenheit denote the same temperature as 35° of the Centigrade.

4. *S. H:* 1849. "Temperate" is marked on Fahrenheit's

thermometer at 56°, find the corresponding number of degrees on the Centigrade.

5. What number of degrees of Fahrenheit corresponds with 4° below zero of the Centigrade?

6. What number of degrees of the Centigrade corresponds with zero of Fahrenheit?

7. *S. H:* 1850. At what temperature will Fahrenheit and the Centigrade show the same number of degrees?

8. *S. H:* 1849. The temperature at one place is 35° by the Centigrade, and at another 64° by Fahrenheit. Find the difference of these temperatures by Fahrenheit.

9. Find the temperature at which Fahrenheit shows as many degrees above zero as the Centigrade does below zero.

10. A Fahrenheit's thermometer is not graduated higher than the boiling point, and the length of the scale is one-fourth more than the distance between the freezing and boiling points. Find the degrees of the Centigrade which correspond with the lowest graduation on the instrument described.

APPENDIX II.

ANSWERS; AND HINTS FOR THE SOLUTION OF SOME OF THE QUESTIONS.

i. DENSITIES.—ART. 7.

EXAMPLE. ANSWERS.

1. The bodies being of the same size, they will be of the same density if their weights be equal.

2. As 5 : 4. [For the weight of 40 cubic inches of the second substance being 20×1 lbs., or 20 lbs., $\therefore \frac{\text{Density of the first substance}}{\text{Density of the second substance}} = \frac{\text{Weight of 40 cub. inches of the first substance}}{\text{Weight of the same bulk of the second substance}} = \frac{25}{20} = \frac{5}{4}$.]

3. As 7 : 10.

4. As 1 : 7. [For $\frac{\text{Density of Water}}{\text{Density of Zinc}} = \frac{\text{Weight of a cub. foot of water}}{\text{Weight of same bulk of zinc}} = \frac{1000}{7000} = \frac{1}{7}$.]

5. As 93 : 100.

6. As 1 : 8. [For the definition of "Bulk" see Art. 66.]

7. Bulk of the former substance : bulk of the latter substance :: 9 : 4.

[$\frac{\text{Density of the first substance}}{\text{Density of the second substance}} = \frac{4 \text{ lbs.}}{9 \text{ lbs.}} = \frac{4}{9}$. And since if the weight of a substance be given, its magnitude (M) must be diminished in the same proportion as its density (D) is increased, therefore $M \propto \frac{1}{D}$, and $\therefore$ any bulk of the first substance : a bulk, equal to it in weight, of the second substance :: Density of the second substance : Density of the first substance :: 9 : 4.]

EXAMPLE. ANSWERS.

8. Five-sevenths of a cubic inch.

[$\frac{\text{Density of the former substance}}{\text{Density of the latter substance}} = \frac{\text{Weight of a cubic inch of the former.}}{\text{Weight of a cubic inch of the latter.}}$

And if x = the number of cubic inches required, the above equation becomes, by substitution, $\frac{1}{2} = \frac{280}{\frac{400}{x}} = \frac{28}{40}\,x \;\therefore\; x = \frac{20}{28} = \frac{5}{7}$.]

9. The Density of the former body : Density of the latter :: 12 : 1.

10. One-eighth of a cubic foot.

11. 48 lbs.

12. Quantity of matter in the wood : quantity of matter in the iron :: 2 : 5.

Density of the wood : density of the iron :: 2 : 15.

13. 42, 24, 32, 21, are four such numbers.

[The weights of cylinders of the substances, which are each 1 inch long, are in ounces $\frac{2}{3}$, $\frac{4}{4\frac{1}{2}}$, $\frac{3\frac{1}{2}}{6}$; and a cylinder of water of the same size will therefore weigh $2 \times \frac{2}{3}$ oz. Wherefore the densities of water and of the three cylinders will be as the quantities $2 \times \frac{2}{3}$, $\frac{2}{3}$, $\frac{4}{4\frac{1}{2}}$, $\frac{3\frac{1}{2}}{6}$; or as $\frac{4}{3}$, $\frac{2}{3}$, $\frac{8}{9}$, $\frac{7}{12}$, (and bringing these fractions to the common denominator 36,) as $\frac{48}{36}$, $\frac{24}{36}$, $\frac{32}{36}$, $\frac{21}{36}$; which are to one another as the whole numbers, 48, 24, 32, 21.]

ii. LEVERS.—ARTS. 17–27.

EXAMPLE. ANSWERS.

1. 40. [For, Art. 20, number of lbs. required : 56 :: 10 : 14, whence the answer is obtained.]

2. 2 feet 11 inches. [By Art. 20.]

3. 4 feet 8 inches. [The balancing forces being *weights*, they act in the same direction, and the lever is therefore of the first kind.]

4. 3 st. 12 lbs.

5. *48 lbs.* and 64 lbs.

EXAMPLE. ANSWERS.

6. The fulcrum is 6 feet from the weight of 6 lbs.

[If x be the feet that the fulcrum is from the weight of 6 lbs., by Art. 20, $\frac{x}{10-x} = \frac{9}{6}$.]

7. The fulcrum is 4 inches from the greater weight.

8. 5 lbs. [First find, by Art. 20, the second weight.]

9. 2 lbs. acting at an arm 4 feet long.

10. The fulcrum is 18 inches from the greater force.

11. At a distance from the fulcrum of one-fourth the length of the lever.

12. 4 feet.

13. The distance required is $1\frac{1}{2}$ inch. The pressure on the fulcrum is 21 lbs., and it acts in the same direction as the greater of the forces.

14. 128. [Consider the question as a case of a lever kept at rest round the shoulder of the man who is the nearer to the weight.]

15. 7 lbs. [Suppose the lever kept at rest round the prop (as fulcrum) which is $3\frac{1}{2}$ feet from the weight, the resistance of the other prop being the power.]

16. At 20 inches from either extremity of the lever.

17. If the weight be placed 1 foot from an end of the lever, the pressure on the other end will be 8 lbs.

18. The weights are 8 lbs. and 24 lbs., acting at arms of 9 and 3 inches in length respectively.

19. 10 inches. [The Power is the greater, and the Weight the lesser force. The length of the longer arm being taken to be x inches, the equation $\frac{8}{10} = \frac{x-2}{x}$ will give its value.]

20. 1·2 feet from the end to which the weight of 4 lbs. is attached.

[Since $1\frac{1}{2}$ inches weigh 3 oz., $\therefore$ 1 inch weighs 2 oz., and the whole cylinder 48×2 oz. or 6 lbs. Supposing the matter of the cylinder to be collected at its middle point, and taking x the distance in feet of the fulcrum from the weight of 4 lbs., x may be determined from the equation $\frac{x}{2-x} = \frac{6 \text{ lbs.}}{4 \text{ lbs.}} = \frac{3}{2}$.]

EXAMPLE. ANSWERS.

21. The fulcrum must be moved eight-tenths of an inch towards the other extremity of the cylinder.

22. This will be the case if the forces act in the directions of the arms at which they are respectively applied.

23. The arms of the lever are as 2 to 3.

[Let x be the ounces in the weight, AB the lever, C the fulcrum, CA the shorter arm, then

$\frac{x \text{ oz.}}{8 \text{ oz.}} = \frac{CB}{CA}$, and $\frac{x}{18} = \frac{CA}{CB}$ $\therefore \left(\frac{CA}{CB}\right)^2 = \frac{8}{18} = \frac{4}{9}$, and $\frac{CA}{CB} = \frac{2}{3}$.]

24. $P = W$, and is applied at the same point.

25. The second. [The fulcrum is the axle of the wheel, the Power the force exerted at the extremities of the handles, and the Weight is the load together with the weight of the body and handles of the wheelbarrow.]

26. Proof. [(The figure will easily be drawn from the following description.) Let P and Q be the weights which, when suspended from A and B, balance round the fulcrum C; M and N, the points in which vertical lines through A and B meet the horizontal line DCE, drawn through C. Suppose the lever to come into any position, such as $A'CB'$, and m and n to be the points in which vertical lines through A' and B' meet DCE; then, by similar triangles, $\frac{Cm}{Cn} = \frac{CA'}{CB'} = \frac{CA}{CB} = \frac{CM}{CN} = \frac{Q}{P}$; but Cm and Cn are perpendiculars from C on the lines of action of P and Q when suspended at A' and B'; wherefore P and Q balance on $A'CB'$.]

27. The fulcrum is $13\frac{1}{3}$ inches from that end of the rod first mentioned which is not joined to the other rod.

28. 224.

29. 6 inches and 10 inches.

30. The weights are 13 lbs. and 18 lbs., and the lengths of the arms at which they act are 16·2 inches and 11·7 inches respectively.

31. (1) 11 lbs.; (2) $5\frac{1}{2}$ inches.

32. The forces are 14 lbs. and 30 lbs.; the distance required is 8 feet 9 inches.

33. The fulcrum is 21·6 inches from that extremity of the *cylinder to* which the weight is attached.

EXAMPLE. ANSWERS.

34. $\frac{15}{2}\sqrt{3}$ inches. [Let AB, BC, be the lengths of the platina and silver cylinders, D and E their middle points, $BC = x$ inches; then

$$\frac{BD}{BE}\left(=\frac{2\,BD}{2\,BE}\right) = \frac{\text{weight of cylinder } BC}{\text{weight of cylinder } AB}$$

$$= \frac{\text{weight of silver cyl. length } BC}{\text{weight of platina cyl. length } BC} \times \frac{\text{weight of pl. cyl. } BC}{\text{weight of pl. cyl. } AB}$$

$$\therefore \frac{9}{x} = \frac{48}{100}\cdot\frac{x}{9}, \text{ whence } x \text{ may be determined.}]$$

35. 81.

36. 16 inches. [If AB be the length of the cylinder, and BC (x inches) that of the bore, then

$$\frac{\frac{1}{2}(24-x)}{\frac{1}{2}x} = \frac{\text{weight of } BC}{\text{weight of } AC} = \frac{\text{weight of } BC \text{ before boring}}{\text{weight of } AC}$$

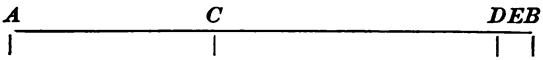
$$\times \frac{\text{weight of } BC \text{ after boring}}{\text{weight of } BC \text{ before boring}} = \frac{BC}{AC}\cdot\frac{1}{4} = \frac{1}{4}\cdot\frac{x}{24-x};$$

whence x can be found.]

37. 75. [Take n = the number of shavings. The question is to be solved in the same manner as the last.]

38. (1) The longer arm preponderates; (2) $\frac{2}{3}$ of an inch is to be cut off the longer and attached to the extremity of the shorter arm.

[To solve the latter question, take AC, CB, the two cylinders; CD that length of the second cylinder which will balance the first; BE (x inches) the part to be taken from the second cylinder and attached to the first in order to produce equilibrium round C.



Then, since AC and CD balance, $AC : CD ::$ weight of CD : weight of $AC :: CD.\frac{2}{3} : AC.\frac{3}{2}$; $\therefore CD^2 = AC^2.\frac{3}{2}.\frac{3}{2}$; $\therefore CD = AC.\frac{3}{2}$ $= 8.\frac{3}{2}$ inches $= 12$ inches; and $\therefore DB$ is one inch.

Again, since AC balances CD, EB when added at A will balance DE.

$\therefore \frac{\text{weight of } EB}{\text{weight of } DE}\left(=\frac{EB}{DB-EB}\right) = \frac{CA+\frac{1}{2}EB}{CD+\frac{1}{2}DE}$; and if EB be x inches, $\therefore \frac{x}{1-x} = \frac{8+\frac{1}{2}x}{12+\frac{1}{2}(1-x)} = \frac{16+x}{25-x}$; whence $x = \frac{2}{5}$.]

EXAMPLE. ANSWERS.

39. In the *first* the Pressure is $W + P$, where P may be of any magnitude:

It therefore must be greater than W, and may be as great as any weight whatever.

In the *second* the Pressure is $W - P$, where P is less than W:

It therefore may be as small as any weight whatever, and cannot be so great as W.

In the *third* the Pressure is $P - W$, where P is greater than W:

It therefore may be as small as any weight whatever, and as great as any weight whatever.

40. [(The figure may be drawn from the following description.) Let CA be the rod, moveable round C; AA' the line in which the force P acts; CM a perpendicular from C on AA'. Take $CR : CM :: P : Q$: and with radius CR describe a circle round C as center. Bisect CA in O; and with O as center and radius OA describe a semicircle, on the same side of CA as Q acts, cutting the former circle in N. Join AN, CN. It may be proved that ANC is a right angle, and that AN is the line in which Q acts.]

41. The tension of one of the strings : weight of the rod $:: 1 : \sqrt{3}$. [Let AB be the rod, C the fixed point; from A draw AE perpendicular to CB. Consider the equilibrium as sustained upon a lever AB moveable round the fulcrum A. AE is the perpendicular on BC, the line in which the force at B is exercised; the weight of the rod acts at D, its middle point.]

42. 3 : 4. [Take the figure to Art. 25. By hypothesis, $\angle CAM = \angle CBN$, and the angles at M and N being also right angles, it may be shown that CAM and CBN are similar triangles, and thence that $CM : CN :: CA : CB$; whence the answer may be obtained.]

iii. COMPOSITION AND RESOLUTION OF FORCE. ARTS. 28–32.

EXAMPLE. ANSWERS.

1. A force of 11 oz. acting in the same direction as the latter set of forces.

2. The resultant being equal to the sum of the component *forces*, they must act in the same line.

3. One-third of four right angles.

EXAMPLE. ANSWERS.

4. If P be one of the forces, the resultant is $P\sqrt{2}$, and its direction bisects the angle at which the forces are inclined to each other.

5. CB. [Since AB, BC, CA acting at A would keep it at rest, therefore BC would counteract the resultant of AB and CA.]

6. AC.

7. 9 lbs. and 12 lbs. [Take $3x$ and $4x$ to represent the numbers of pounds in the forces. Since they act at right angles and produce a resultant of 15 lbs., $\therefore (3x)^2 \times (4x)^2 = 15^2$; whence $x = 3$.]

8. 13 lbs.

9. [Produce AC and BC to A' and B', making $CA' = CA$ and $CB' = BC$. Join $B'A'$. Then CB' and $B'A'$ applied at C would produce a resultant CA'. Therefore the three forces AC, CB', $B'A'$ applied at C would produce a resultant $AC + CA'$. If $\therefore CA'$ be produced to D so that $A'D = CA'$, CD will represent the resultant required. The figure will easily be drawn from the foregoing description.]

10. To prove this, divide the figure into two triangles by joining A and D.

11. Let ABC be the triangle whose sides (taken in order) are in the same proportion as the forces P, Q, and R which keep the point at rest. Then $AB : BC : CA :: P : Q : R :: 2P : 2Q : 2R$; wherefore $2P$, $2Q$, $2R$ will keep the point at rest.

12. Join BC, and from A draw AD equal to CB and parallel to it; AD is the line required.

13. $2BC$.

14. AD. [Complete the parallelogram ($ABECA$) of which AB and AC are the sides. Its diagonals will be bisected in D. From this construction the answer will easily be obtained.]

15. The other force is $10\sqrt{2}$ lbs.; and the further effect it has on the lever is to produce a pressure of 10 lbs. on the fulcrum in the direction of the arm at which it acts. [Let AC be the arm perpendicularly to which the force of 10 lbs. acts, C being the fulcrum; AD making half a right angle with AC; CM a perpendicular from C on AD: $x =$ the pounds in the force that acts in AD.

EXAMPLE. ANSWERS.

$$\text{Then } \frac{x}{10} = \frac{CA}{CN} = \frac{CA}{\frac{CA}{\sqrt{2}}} := \sqrt{2}, \text{ and } \therefore x = 10\sqrt{2}.$$

From M draw Mm perpendicular to AC. Taking AM to represent the force in AD, it may be resolved into Am and mM, of which the latter only can have any effect in supplying the place of the original force of 10 lbs. The other, Am, is a pressure in the direction AC, and it may be proved to be 10 lbs. in magnitude.]

iv. VELOCITIES. ART. 43.

EXAMPLE. ANSWERS.

1. 44.

2. 50·5.

3. The velocity of the first body is 64 times that of the second.

4. 39.

5. 13.

6. 5·75.

v. MECHANICAL POWERS. ARTS. 34—45.

EXAMPLE. ANSWERS.

1. The Power $= \sqrt{(16^2 - 3^2)} = \sqrt{(256 - 9)} = \sqrt{247} = 15{\cdot}71$ lbs.

2. 28 lbs. [Take T the tension of the string, in lbs.; then, x being the lbs. required, and h the common height of the planes in inches,

$$\frac{x}{T} \cdot \frac{T}{42} = \frac{16}{h} \cdot \frac{h}{24} :$$—whence x is found.]

3. 14 lbs. [If T lbs. be the tension of the string, and x lbs. the power, then, Arts. 40 and 42,

$$\frac{T}{56} = \frac{1}{2^2}, \text{ and } \frac{x}{T} = \frac{\text{length of the plane}}{\text{its height}} = 2.]$$

4. 32,000. [Drawing a perpendicular from the fulcrum of the lever *on the direction* of the string attached to the weight that rests on the plane, *that perpendicular* : length of the arm :: height of the plane : its length.]

EXAMPLE. ANSWERS.

5. 2 : 3.

6. The fulcrum is one-ninth of the whole length of the lever, measured from the point to which the string is attached that passes round the pulley from which the weight is suspended.

7. The greatest value is $\frac{1}{2}$, the least value is $\frac{1}{3}$.

8. The Power applied to the circumference of the wheel : the weight sustained on the plane

$$: \frac{\text{radius of the axle}}{\text{radius of the wheel}} :: \frac{\text{length of the plane}}{\text{its height.}}.$$

9. 20 lbs.

10. $\frac{7}{8}$ *W*.

11. 120 lbs.

12. (1), 112 inches; (2), 112 inches; (3), 89·6 inches.

13. 4 cwt. [See Arts. 35 and 42.]

vi. CENTER OF GRAVITY. ARTS. 46—54.

EXAMPLE. ANSWERS.

1. It is between the second weight and the third, and is 8 inches from the latter.

2. If the C. G. be not in the line, the body will nevertheless balance upon the line when its C. G. is brought into the vertical plane that passes through the line: and there are two such positions, one when the C. G. is above, and the other when it is below the line. But if the body be brought into any other position than these two, it will have a tendency to move round the line (Art. 47), unless the C. G. be actually in the line.

3. Proceed as in Art. 52.

4. Let AB, AC be the sides. Bisect them respectively in D and E; join DE, and divide it in G, so that $DG : DE :: AC : AB + AC : G$ is the C. G. required.

EXAMPLE. ANSWERS.

5. [Take the figure to Art. 52, omitting the line bef; and join EF. Then since $BE : BA :: 1 : 2 :: BF : FC$, $\therefore$, Euclid vi. 2, EF is parallel to FC; whence BEF, BAC may be proved to be equiangular triangles, and $\therefore EF : FB :: AC : CB$, and $\therefore EF : AC :: FB : CB :: 1 : 2$.

Again, EF being parallel to AC, EFG and CAG may be shown to be equiangular triangles, and that $AG : GF :: AC : FE :: 1 : 2$; $\therefore GF = 2AG$, and $\therefore AG = \frac{2}{3} AF$.]

6. Let AB be the given side, G the given C. G. Bisect AB in C; join CG and produce it to D, making $GD = 2\,CG$. Join AD, DB : ADB may be proved to be the triangle required.

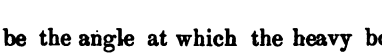
7. Let ABC be the triangle. Bisect AB in D, join DC, and take $DG = \frac{1}{3} DC$. The C. G. required is G; and it may be proved (Art. 52) to be the C. G. of the triangle also.

8. Let weights of 1 lb., 2 lbs., 4 lbs. be placed at the angles A, B, and C respectively of the triangle ABC. Take D in AB such that $AD : AB :: 2 : 3$. Join DC, and take G in it such that $DG : CD :: 4 : 7$; G may be proved to be the C.G. required.

9. Let AB be the given horizontal base; through B draw BC at right angles to AB, and let D be any point in it. Join AD and produce it to E, making $DE = AD$. Join BE. It may be proved that ABE is such a triangle as is required to be drawn.

10. Let A be the angle at which the heavy body is placed. Bisect BC in D and AB in E. Join AD and CE, intersecting in F. In AF take the point G such that $AG : AF :: 1 : 8$; G may be proved to be the C. G. required.

11. [Divide AB in F so that $AF : AB :: Q : P + Q$; then F is the C.G. of P and Q.



Divide FC in a point (G) such that $FG : FC :: R : (P + Q) + R$; G is the C. G. required. Now $(P + Q) . AF = Q . AB$, $\therefore$ $(P + Q) . (AF + OA) = Q . AB + (P + Q) . OA$, or $(P + Q) . OF = P . OA + Q . OB$. Similarly, supposing P and Q collected at F their C. G., and R to be at C, $(P + Q + R) . OG = P + Q . OF + R . OC$, $\therefore$ $(P + Q + R) . OG = P . OA + Q . OB + R . OC$.]

12. PROOF. [Let ABC be the triangle; and let b c, parallel to BC, divide the triangle into two equal parts. Join A with D the bisection of

EXAMPLE. ANSWERS.

BC; it may be shown that the line will bisect bc in the point d where it cuts it. Bisect AB in E and Ab in e, and join BE and be, cutting AD in G and g: G and g are the C. Gs. of the Δs. ABC, Abc.

Now the C. G. of Δ Abc, and that of Δ ABC, are both in the line AD. And since the quadrilateral $bBCc$ and the Δ Abc have for their common C. G. the point G, the C. G. of the quadrilateral must be in the same line as gG produced. Let H be a point taken in GD, such that $GH : Gg ::$ weight of Δ Abc : weight of $bBCc :: 1 : 1$; it may be proved that H is the point required.]

13. [Let $ABCD$ be the parallelogram, and E the point in which its diagonals intersect. Let ac, parallel to AC, cut off the Δ Bac whose area is one-third of the whole parallelogram. The diagonal BD will bisect ac in the point e where it cuts it. Bisect Bc in f, and join af, cutting Be in g; g is the C. G. of the Δ Bac. It may be proved by the same mode of reasoning as that employed in solving the last question, that if H be a point in ED such that $EH : Eg :: 1 : 2$, H is the C. G. required.]

14. 10 inches.

15. [Through E draw FEH parallel to AD or BC, bisecting AB in F and CD in H. Bisect BE in K and join AK cutting FE in the point G, which will be the C. G. of the Δ AEB. In EH take a point G', such that $EG' : EG :: 1 : 3$; it may be proved that G' is the point required.]

16. If EF be drawn from E perpendicular to AB, and a point G be taken in it such that EG is equal to one-twelfth of a side of the square, G is the C.G. required.

17. This may be proved by showing that the vertical from the $C.G.$ of the side which is a rhombus, will cut the horizontal base of the rhombus. The solid may be considered to be made up of material planes parallel to such a side.

HYDROSTATICS.

vii. PRESSURE OF NON-ELASTIC FLUIDS.

ARTS. 58—65.

EXAMPLE. ANSWERS.

1. The pressure on every point of the triangle being the same (Art. 59), the reasoning of Art. 52 is applicable.

2. 36 lbs.

EXAMPLE. ANSWERS.

3. The pressures are (Art. 62) as $4\times3\times4 : 6\times4\times2$ (which are as 1 : 1), and are therefore equal.

4. The hand being kept in the vessel with a force equal to the weight of the fluid displaced will produce the same effect as a weight of fluid equal to that displaced. The pressure on the base of the vessel will therefore be increased by the weight of that quantity of the fluid. The scale containing the vessel will consequently descend.

5. In both cases the additional pressure will be equal to the weight of the solid.

viii. SPECIFIC GRAVITIES. ARTS. 67—75.

EXAMPLE. ANSWERS.

1. 7·776.

2. 2·56, and 1·327.

3. 11 : 10. [Here, since $W = MS$, and W is the same in each case, $\therefore MS \propto 1$ and $S \propto \frac{1}{M}$; wherefore S. G. of the first : S. G. of the second fluid :: 22 : 20.]

4. 8. [Since $W = MS$, $\therefore S \propto \frac{W}{M}$.]

5. 6·25 times.

6. 40 feet. [Take x the length in feet of an edge, then

$$x^3 \times 1\cdot12 : 1 :: 2000 \times 20 \times 112 \times 16 : 1000;$$

$$\therefore x^3 = \frac{2000 \times 20 \times 112 \times 16}{1000 \times 1\cdot12}, = (40)^3, \text{ by reduction.}]$$

7. 77 lbs. [Since $W = MS$, and by hypothesis M is constant here, $\therefore W \propto S$, and the weight required $= \frac{91 \times \cdot66}{\cdot78}$ lbs. $= \frac{91 \times 66}{78}$ lbs.; which by reduction comes to 7×11, or 77 lbs.]

8. 51·2 cubic inches: S. G. of the compound metal = 8.

[Since $MS = W = M'S' \therefore S''$, the S. G. of the compound, which

$$= \frac{MS + M'S'}{M + M'}, = \frac{2\,MS}{M + \frac{MS}{S'}} = \frac{2\,SS'}{S' + S} = \frac{2 \times 7\cdot2 \times 9}{7\cdot2 + 9}$$

$$= \frac{2 \times 8 \times \cdot9 \times 9}{16\cdot2} = \frac{8 \times \cdot9 \times 9}{8\cdot1} = 8.]$$

EXAMPLE. ANSWERS.

9. 1·2 times.

10. 48·6 lbs.

11. 1·02.

12. ·955.

13. 1·74.

14. 1·05.

15. 1·08.

16. The volumes employed are equal. [Take M and $r\,M$ to represent the volumes, S and S' the S. Gs. It may be proved that $r = 1$.]

17. The formula may be proved by the method employed in Art. 70.

18. ·904 nearly.

19. 112 : 109.

20. 1·125. [The S. G. required being that of the brine, the answer may be found by determining the S. G. of a pint of water in which two oz. of salt have been dissolved without increasing its bulk.]

21. 1·5.

22. If bulk had been neither gained nor lost, the S. G. of the mixture would have been 1·31 ...; the S. G. of the mixture being less than this, bulk must have been gained.

23. 1·0101.

24. 10. [Bulk of water equal in size to the substance : 1 pint :: $\frac{1}{2}$ in. : 4 in.; $\therefore$ the weight of such a bulk of water $= \frac{1}{4} \times 1$ lb. $= 2$ oz.]

25. μ is the weight, in ounces, of a bulk of water of the same size as the body whose weight is W oz.

[Since (see foot note to Art. 67) $\frac{\text{weight of } \textit{any} \text{ bulk of the substance}}{\text{weight of an } \textit{equal} \text{ bulk of water}} = \sigma$

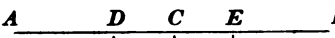
$= \frac{W}{\mu}$ by the question, $= \frac{\text{weight (in oz.) of a } \textit{certain} \text{ bulk of the substance}}{\mu}$

$\therefore$ μ must be the weight, in ounces, of a bulk of water equal to that of the substance whose weight is W oz.]

26. £14,492 10*s*. [The tunnel contains $880 \cdot 3 \cdot 3.12$ cubic feet; of which the weight in tons is $\frac{880 \cdot 3 \cdot 8 \cdot 12 \cdot 1{\cdot}4 \cdot 1000}{16 \cdot 112 \cdot 20} = 11 \cdot 3 \cdot 3 \cdot 160$.

EXAMPLE. ANSWERS.

Again, if AB be the length of the tunnel, C its middle point, D and E points equidistant from C, it may be proved that $AD + AE = 2\,CE$; whence it appears that the space through which the excavated earth is carried is the same as if it were all collected at C. Wherefore the cost of removing the earth is



$$440 \times 11.3.3.100 \cdot \frac{3}{4.12.20}\,£ = \frac{1}{8} \cdot 11.11.9.£100 = £13{,}612.10s.$$

And $\therefore$ the whole cost of making the tunnel = £14,492.10s.]

27. S. G. of platina = 2·756, S. G. of water = 0·128.

[If σ_p, σ_i, 1 be the S. Gs. when that of water is taken to be 1, S_p, 1, S_w be the S. Gs. when that of iron is taken to be 1, $S_p : \sigma_p :: 1 : \sigma_i$, and $S_w : 1 :: 1 : \sigma_i$,

$\therefore S_p = \frac{\sigma_p}{\sigma_i}$, and $S_w = \frac{1}{\sigma_i}$.]

28. $8\frac{1}{100}$ lbs.

29. 3 : 2.

[S. G. of fluid : S. G. of solid :: bulk of whole body : bulk of the part immersed :: 6 : 4 :: 3 : 2.]

30. ·24.

31. 9 cubic inches.

32. 1·152.

33. S. G. of solid : S. G. of fluid :: 3 : 5. And on the solid being shortened four inches, it will float with 2·4 inches above the surface.

34. 12 lbs.

35. 2 oz. [The weight of the block + 1 oz. is equal to the weight of a bulk of the fluid of the same magnitude as the block.]

36. 1·2.

37. $\frac{8}{100}$ of an inch.

38. 375. [The weight of the nails *plus* that of the wood is 1000 oz.]

39. 1 lb. [If x be the lbs. in the weight required, $3 + x : 3$ :: weight of a bulk of the fluid : weight of an equal bulk of the solid.]

40. ·9375.

EXAMPLE. ANSWERS.

41. 6 lbs. [The second body, being of half the size and twice the S. G. of the other, is equal to it in weight. And if x be the weight of the substance in lbs., $2x : x ::$ S. G. of fluid : S. G. of solid $:: x : x - 3$.]

42. 6 lbs.

43. (1) Two-fifths of the wood must be hollowed out. (2) The S. G. of the wood is 1·25. [(1) By the question, the weight of an equal bulk of water is 8 lbs. Let $10x$ lbs. of the wood be left when the requisite portion is hollowed out. Then, by the question $(10x + 2)$ lbs. = weight of a bulk of water of the same size as the original piece of wood, = 8 lbs. $\therefore x = \frac{3}{5}$, $\therefore \frac{2}{5}$ in weight, and $\therefore$ in bulk also, of the wood has been taken away.]

44. $2\frac{2}{3}$. [The S. G. of the wood $= \frac{2}{3}$, and if M be the number of cubic inches in the wood, and s the S. G. required, the equation $M\frac{2}{3} + \frac{M}{8}s = M$ will give the answer.]

45. 240 oz. [The equilibrium will remain if the cork be removed and its place supplied by a bulk of water of the same size as the fluid the cork displaced. The additional pressure on the base will, by Art. 71, be the weight of that water, which = the weight of the cork.]

46. 18½ lbs.

47. 25,200 lbs.

48. The S. G. of the lead is 11; that of the compound $13\frac{14}{15}$. [The weight of a bulk of water the same size as the gold $= \frac{95}{19}$ oz. = 5 oz.; $\therefore$ the gold (and so also the lead) weighs 90 oz. in water; $\therefore$ the lead loses 9 oz. by being weighed in water.]

49. $1\frac{125}{152}$. [First, to solve the question by the application of formulæ. Taking the second formula of Art. 70, $\frac{2}{\sigma''} = \frac{1}{8} + \frac{1}{19}$. And, Art. 68, weight of an equal bulk of water = weight of the compound $\times \frac{1}{\sigma''} = \left(\frac{1}{8} + \frac{1}{19}\right)$ lbs. = weight lost by the compound in water (Art. 72); $\therefore$ weight of the compound in water $= \left\{2 - \left(\frac{1}{8} + \frac{1}{19}\right)\right\}$ lbs. $= 1 + \left\{1 - \left(\frac{1}{8} + \frac{1}{19}\right)\right\}$, lbs. $= 1\frac{125}{152}$ lbs.

EXAMPLE. ANSWERS.

Next, to solve the question by the application of an elementary principle. Since the masses of iron and of gold lose no bulk by being mixed, ∴ the whole weight lost by the compound will be equal to the sum of the weights lost by the pound of iron and the pound of gold when weighed by themselves. Now the pound of iron loses the weight of an equal bulk of water, which is $1 \times \frac{1}{8}$ lbs.; so the pound of gold loses $1 \times \frac{1}{19}$ lbs.; wherefore the compound weighs $\left(2 - \frac{1}{8} - \frac{1}{19}\right)$ lbs. $= 1\frac{125}{152}$ lbs.]

50. ·625. [The weight of an equal bulk of water − weight of the wood equal 3 lbs.]

51. $7\frac{115}{139}$.

52. $\frac{s' - s''}{s' - s}$; where s, s' are the S. Gs. of the lighter and heavier fluid respectively, and s'' is that of the solid.

[By the same reasoning as that employed in Art. 65, it may be proved that $(M + N)s'' = Ms + Ns'$, = (by adding and subtracting Ms') $Ms + Ns' - Ms' + Ms' = (M + N)s' - M(s' - s)$; ∴ $M(s' - s) = (M + N)(s' - s'')$.]

53. Volume of the lighter fluid : volume of the heavier fluid :: $(a + b)s - (a - b)s' : 2as$; where s and s' are the S. G.s of the lighter and the heavier fluids.

[Let volume of the lighter $(m) \div$ volume of the heavier $(m') = x$. S. G. of the mixture, Art. 69, $= \frac{ms + m's'}{m + m'} = \frac{xs + s'}{x + 1}$. The second mixture contains a volume $\frac{1}{2}(m + m')$ of the first mixture, and the same volume of the lighter fluid, wherefore its S. G. is

$$\frac{\frac{1}{2}(m + m')\frac{xs + s'}{x + 1} + \frac{1}{2}(m + m')s}{m + m'} = \frac{1}{2}\left(\frac{xs + s'}{x + 1} + s\right);$$

And since the fluid displaced is in each case equal to the weight of the body,

$$\therefore \frac{a}{a + b} = \frac{\text{S. G. of 2nd mixture}}{\text{S. G. of 1st mixture}} = \frac{1}{2}\left\{1 + \frac{(x + 1)s}{xs + s'}\right\}$$

$$\therefore \frac{2a}{a + b} - 1 \left(\text{or } \frac{a - b}{a + b}\right) = \frac{(x + 1)s}{xs + s'}; \text{ whence } x = \frac{(a + b)s - (a - b)s'}{2as}.]$$

54. $1\frac{1}{8}$ inches.

55. $W \div w$ is less than $W' \div w'$.

[$\frac{W}{W - w}$, the S. G. of the first, is greater than $\frac{W'}{W' - w'}$, the S. G. of *the second.* Hence the answer may be found.]

EXAMPLE. ANSWERS.

56. $n \,.\, (r - q) \div (p - q)$. [If x be the ounces required, since x oz. of gold would weigh in water $x \,.\, \frac{p}{n}$, and $(n - x)$ oz. of silver would weigh $(n - x)\,\frac{q}{n}$, $\quad \therefore x\,\frac{p}{n} + (n - x)\,\frac{q}{n} = r$.]

57. $R \,.\, \left(\frac{r}{R} - \frac{q}{Q}\right) \div \left(\frac{p}{P} - \frac{q}{Q}\right)$.

58. $p \div P$, $r \div R$, $q \div Q$ are in order of magnitude, $p \div P$ being the greatest.

59. $25\frac{1}{2}$. [If x = oz. required, $45 - x$ = oz. of second metal; and if y be the oz. each mass loses by immersion, since 51 oz. of the first loses y by immersion, and 39 oz. of the second loses the same weight, the equation, $x \,.\, \frac{y}{51} + (45 - x)\,\frac{y}{39} = y$ (from which y disappears) will give x.]

60. The effect will be the same as if a bulk of fluid equal in magnitude to the body immersed were added to the fluid in the vessel. The scale will therefore preponderate in which the vessel is placed.

61. The second weight is placed in the diagonal that passes through the former weight, and at a distance of 7 inches from that weight.

62. [The height to which the water rises in the upper cylinder $= x$ inches $= \frac{1}{2}\,(12 \times 12 - 3 \times 27)$ inches; and the pressure on the base $= \frac{1000 \times 54\,(18 + x)}{12 \times 12 \times 12}$ oz. $= \frac{99000}{8 \times 8}$ oz.]

63. The altitudes of the mercury and of the water are 6 inches and 30 inches.

ix. ELASTIC FLUIDS. ARTS. 77—92.

EXAMPLE. ANSWERS.

1. It rises in consequence of its weight, together with that of the gas it contains, being lighter than a quantity of air of the same size as the balloon and the gas; and it rises slower as it gets higher because the density of the air is less and less as the balloon rises from one altitude to another.

2. A length that is just less than 30 inches.

EXAMPLE. ANSWERS.

3. 37·5 feet.

4. The altitude of the water will be 42 inches; and if the mercurial column sink 1 inch, the surface of the water will also sink 1 inch.

5. $234\frac{11}{31}$ inches.

6. 56·16 inches, nearly.

7. It would depress the mercurial column until the pressure produced by the included air (which becomes rarer as the mercurial column falls) and that of the shortened mercurial column together balance the pressure of the external air.

8. (1.) Since air, when the barometer is at 28 inches, exercises a less pressure than when the barometer is at 30 inches, its density (Art. 78) is greater in the latter case than in the former, and therefore the substance that is of the lesser S.G., being of greater bulk than the other, will lose more weight. The substance therefore that is of the greater S.G. will preponderate when the barometer rises to 30 inches. (2.) The seller loses.

9. 21 cwt.;—the compression being supposed to be performed in vacuo, so that the pressure on the piston is not assisted by the pressure of the atmosphere on it externally.

10. 6·2 inches. [Let A be the top of the tube; B C D the points at which the mercury stands at first and after each syringeful of air is introduced. Let $A\,B$ be x inches.

By the question, the pressures of the air introduced, when confined in $A\,C$ and $A\,D$ respectively, sustain 1 inch and 1·8 inches of mercury; and, pressure of the syringeful of air at first in $A\,C$: pressure of twice that quantity if confined in $A\,C$: : 1 : 2 : (Art. 78). Also, pressure of two syringefuls of air in $A\,C$: pressure of the same quantity when in $A\,D$: : $A\,D$: $A\,C$: (Art. 78). Whence it may be shown that $A\,D$: $2\,A\,C$: : 10 : 18.]

A

B

C

D

11. The volume of the air would change with every change of temperature, and would therefore obstruct the *equable* expansion of the fluid contained in the thermometer. If the top of the tube communicated with the atmosphere, the pressure on the surface *of the fluid* would not vary much, the pressure of the atmosphere *changing but little.*

EXAMPLE. ANSWERS.

12. [Let $s\,s'$ be the S.G.s of the lighter and the heavier fluids; s'' the S.G. of the mixture; n the number of divisions immersed when the instrument is placed in the mixture; y, and $100 - y$, the parts, per cent. by weight, of the lighter and heavier fluids in the mixture; M and $M + m$ the volumes of the instrument immersed in the heavier and lighter fluids: then $\frac{M+m}{M} = \frac{s'}{s}$; $\frac{100}{s''} = \frac{y}{s} + \frac{100-y}{s'}$; $\left\{M + \frac{m}{100} \cdot n\right\} s'' = M s'$; whence it may be proved that $n \cdot \frac{m}{M} = y \cdot \frac{m}{M}$.]

13. $n \cdot \frac{m}{100}$.

14. During the eighth descent.

15. One-half.

x. THERMOMETERS. ARTS. 93—94.

EXAMPLE. ANSWERS.

1. 25;—5;—35⅘.
2. 122; 230; 14; —4·9.
3. By the formula, Art. 94, if $C = 35$, $F = 95$.
4. 13½.
5. 24·8.
6. 17⅘ below zero.
7. —40°.
8. 31°.
9. 11⅛°.
10. 25 below zero.

LONDON:
Printed by WILLIAM CLOWES and SONS,
Stamford Street.

CORRECTIONS.

Page 84, Ex. 31.—*After* "cubic inches" *add* "in volume."

„ 86, Ex. 53, line 2.—*After* "two" *add* "given."

„ 92, Ex. 13, line 1.—*For* "42" *read* "48."

„ 94, Ex. 30.—The distance between the points at which the weights act is 27·9 inches.

„ 95, Ex. 38.—There is an error in the working:—

For $\frac{CA + \frac{1}{2}EB}{CD + \frac{1}{2}DE}$ *read* $\frac{CD + \frac{1}{2}DE}{CA + \frac{1}{2}EB}$, and so on: x will be found $= \frac{25}{42}$.

„ 97, Ex. 13.—*Read* "From A draw AD in the direction of BC and $= 2\,BC$; AD is the line required."

„ 98, Ex. 3.—The answer is, "2 cwt. [If T lbs. be the tension of the string attached to the Power, and W lbs. the Weight, then, Arts. 40 and 42,

$$\frac{W}{T} = 2^3, \text{ and } \frac{T}{56} = \frac{\text{height of the plane}}{\text{its length}}.]"$$

„ 102, Ex. 2.—*After* "1·327" *add* "nearly."

„ 103, Ex. 26, line 1.—*For* "880 . 3 . 3 . 12" *read* "880 . 3 . 8 . 12."

„ 104, line 4.—*For* "CE" *read* "AC."

„ 106, Ex. 53, line 2.—*For* "$(a + b)\ s - (a - b)\ s' : 2\,a\,s$" *read* "$a\,(s - s') - b\,.\,(s' + s) : 2\,b\,s$," and correct the value of x accordingly.

„ 107, Ex. 2.—The answer is 1546·875 ounces.

„ 109, Ex. 12, line 7.—*Add* "and $\therefore\ y = n$."